SECTION: VECTOR ALGEBRA & VECTOR CALCULUS

1. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to a is given by : [CSIR June 2011]

(a) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{a^2}$ (b) $\frac{\vec{b}\times(\vec{a}\times\vec{b})}{b^2}$ (c) $\frac{(\vec{a}.\vec{b})\vec{b}}{b^2}$ (d) $\frac{(\vec{b}.\vec{a})\vec{a}}{a^2}$

2. The equation of the plane that is tangent to the surface $xyz = 8$ at the point (1, 2, 4) is : [CSIR Dec. 2011]

(a) $x+2y+4z=12$ (b) $4x+2y+z=12$

(c) $x+4y+2z=12$ (d) $x+y+z=7$

3. A vector perpendicular to any vector that lies on the plane defined by $x+y+z=5$ is: [CSIR June 2012]

(a) $\vec{i}+\vec{j}$ (b) $\vec{j}+\vec{k}$ (c) $\vec{i}+\vec{j}+\vec{k}$ (d) $2\vec{i}+3\vec{j}+5\vec{k}$

4. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}},\frac{b}{\sqrt{3}},\frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is [CSIR Dec. 2012]

(a) $\frac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$ (b) $\frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

(c) $\frac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

5. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120º with respect to $\hat{i}$. The angle between the vectors $\vec{u}=a\hat{i}+b\hat{n}$ and $\vec{v}=a\hat{n}+b\hat{i}$ will be 60º if : [CSIR June 2013]

(a) $b=\sqrt{3}a/2$ (d) $b=2a/\sqrt{3}$ (c) $b=a/2$ (d) $b=a$

6. If $\vec{A}=yz\hat{i}+xy\hat{j}+xy\hat{k}$, then the integral $\oint_C \vec{A}.d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by $x=0, x=a$ and $y=0, y=b$) is : [CSIR Dec. 2013]

(a) $\frac{1}{2}\left(a^3+b^3\right)$ (b) $\pi\left(ab^2+a^2b\right)$ (c) $\pi\left(a^3+b^3\right)$ (d) 0

7. If $\vec{A}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.d\vec{l}$ is : [CSIR June 2014]

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

QUANTA

8. Let $\vec{r}$ be the position vector of any point in three dimensional space and $r = |\vec{r}|$. Then [CSIR Dec. 2014]

(a) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla} \times \vec{r} = \frac{\vec{r}}{r}$ (b) $\vec{\nabla}.\vec{r} = 0$ and $\vec{\nabla}^2 \vec{r} = 0$

(c) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla}^2 \vec{r} = \frac{\vec{r}}{r^2}$ (d) $\vec{\nabla}.\vec{r} = 3$ and $\vec{\nabla} \times \vec{r} = 0$

9. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the positive vector, then the value of the following integral $\iint_S \vec{r}.\hat{n} dS$ is: [GATE 2001]

(a) V (b) 2V (c) 0 (d) 3V

10. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$: [GATE 2001]

(a) The three vectors are orthonormal

(b) The three vectors are linearly independent

(c) The three vectors cannot form a basis in a three-dimensional real vector space

(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combinational of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

11. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2 \vec{A}$ equals : [GATE 2001]

(a) 1 (b) 3 (c) 0 (d) –3

12. A vector $\vec{A} = (5x + 2y)\hat{i} + (3y - z)\hat{j} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value: [GATE 2002]

(a) 4 (b) –4 (c) 8 (d) –8

13. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$ where a and $(a \neq b)$ are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors? [GATE 2002]

(a) $-b\hat{i} + a\hat{j}$ (b) $-a\hat{i} + b\hat{j}$ (c) $-a\hat{i} - b\hat{j}$ (d) $-b\hat{i} - a\hat{j}$

14. The unit vector normal to the surface $3x^2 + 4y = z$ at the ponit (1, 1, 7) is:[GATE 2002]

(a) $(-6\hat{i} + 4\hat{j} + \hat{k})/\sqrt{53}$ (b) $(4\hat{i} + 6\hat{j} - \hat{k})/\sqrt{53}$

(c) $(6\hat{i} + 4\hat{j} - \hat{k})/\sqrt{53}$ (d) $(4\hat{i} + 6\hat{j} + \hat{k})/\sqrt{53}$

Common data for Q.15 and Q.16 **[GATE 2003]**

Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$

QUANTA

15. The surface integral of this vector over the surface of a cube of size and centered at the origin.

(a) 0 (b) 2π (c) $2\pi a^3$ (d) 4π

16. Which one of the following is NOT correct?

(a) Value of line integral of this vector around any closed curve is zero

(b) This vector can be written as the gradient of some scalar function

(c) The line integral of this vector from point P to point Q is independent of the path taken

(d) This vector can represent the magnetic field of some current distribution

17. The curl of the vector $\vec{A} = z\hat{i} + x\hat{j} + y\hat{k}$ is given by : [GATE 2003]

(a) $\hat{i} + \hat{j} + \hat{k}$ (b) $\hat{i} - \hat{j} + \hat{k}$ (c) $\hat{i} + \hat{j} - \hat{k}$ (d) $\hat{i} - \hat{j} - \hat{k}$

18. The two vectors $\vec{P} = \hat{i}, \vec{q} = \left(\hat{i} + \hat{j}\right)/\sqrt{2}$ are : [GATE 2003]

(a) related by a rotation (b) related bya reflection through the xy-p lane

(c) related by an inversion (d) not linearly independent

19. For the function $\phi = x^2 y + xy$ the value of $\left|\vec{\nabla}\phi\right|$ at $x = y = 1$ is : [GATE 2004]

(a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$

20. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{F}\right)$ is : [GATE 2005]

(a) 0 (b) $\hat{i}$ (c) $2\hat{j}$ (d) $3\hat{k}$

21. Given the four vectors, $u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \end{pmatrix}, u_4 = \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix}$, the linearly dependent pair is : [GATE 2005]

(a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4

22. The unit normal to the curve $x^3 y^2 + xy = 17$ at the point (2, 0) is : [GATE 2005]

(a) $\dfrac{\left(\hat{i} + \hat{j}\right)}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$

23. A vector field is defined everywhere as $\vec{F} = \dfrac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y and Z axes, is: [GATE 2007]

(a) L^3 (b) $4L^3$ (c) $8L^3$ (d) $10L^3$

24. If $\vec{r} = x\hat{i} + y\hat{j}$, then : [GATE 2007]

QUANTA

(a) $\vec{\nabla}.\vec{r}=0$ and $\vec{\nabla}|\vec{r}|=\vec{r}$ (b) $\vec{\nabla}.\vec{r}=2$ and $\vec{\nabla}|\vec{r}|=\vec{r}$

(c) $\vec{\nabla}.\vec{r}=2$ and $\vec{\nabla}|\vec{r}|=\frac{\vec{r}}{r}$ (d) $\vec{\nabla}.\vec{r}=3$ and $\vec{\nabla}|\vec{r}|=\frac{\vec{r}}{r}$

25. Consider a vector $\vec{p}=2\hat{i}+3\hat{j}+2\hat{k}$ in the coordinate system $\left(\hat{i},\hat{j},\hat{k}\right)$. The axes are rotated anti-clockwise about the Y axis by an angle 60°. The vector $\vec{p}$ in the rotated coordinate system $\left(\hat{i}',\hat{j}',\hat{k}'\right)$ is : [GATE 2007]

(a) $\left(1-\sqrt{3}\right)\hat{i}'+3\hat{j}'+\left(1+\sqrt{3}\right)\hat{k}'$ (b) $\left(1+\sqrt{3}\right)\hat{i}'+3\hat{j}'+\left(1-\sqrt{3}\right)\hat{k}'$

(c) $\left(1-\sqrt{3}\right)\hat{i}'+\left(3+\sqrt{3}\right)\hat{j}'+2\hat{k}'$ (d) $\left(1-\sqrt{3}\right)\hat{i}'+\left(3-\sqrt{3}\right)\hat{j}'+2\hat{k}'$

26. The curl of the vector field is $\vec{F}$ is $2\hat{x}$. Identity the appropriate vector field $\vec{F}$ from the choices given below: [GATE 2003]

(a) $\vec{F}=2z\hat{x}+3z\hat{y}+5y\hat{z}$ (b) $\vec{F}=3z\hat{y}+5y\hat{z}$

(c) $\vec{F}=3x\hat{y}+5y\hat{z}$ (d) $\vec{F}=2x\hat{y}+5y\hat{z}$

27. The value of the contour integral $\left|\int_C \vec{r}\times d\vec{\theta}\right|$, for a circle C o f radius r with centre at the origin is : [GATE 2009]

(a) $2\pi r$ (b) $\frac{r^2}{2}$ (c) πr^2 (d) r

28. An electrostatic field $\vec{E}$ exists in a given region R. Choose the wrong statement:

(a) Circulation of $\vec{E}$ is zero [GATE 2009]

(b) $\vec{E}$ can always be expressed as the gradient of a scalar field

(c) The potential difference between any two arbitary points in the region R is zero

(d) The work done in a closed path lying entirely in R is zero

29. Consider the set of vectors in three-dimensional real vector space $R^3, S=\{(1,1,1),(1,-1,1)$ $(1,1,-1)\}$. Whch of the following statement is true? [GATE 2009]

(a) S is not a linearly independent set (b) S is a basis for R^3

(c) The vectors in S are orthogonal

(d) An orthogonal set of vectors cannot be generated from S

30. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that : [GATE 2011]

QUANTA

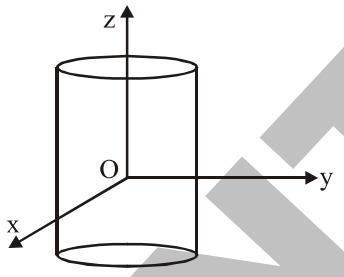
(a) $\vec{\nabla}\times\vec{F}=0$ (b) $\vec{\nabla}.\vec{F}=0$ (c) $\vec{\nabla}V=0$ (d) $\nabla^2V=0$

31. The unit vector normal to the surface $x^2+y^2-z=1$ at the point P(1, 1, 1) is :

[GATE 2011]

(a) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

32. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$ be the position vector and a unit vector normal to the surface. The surface integral $\int_s \vec{r}.\hat{n}ds$ over the closed surface of the cylinder is: [GATE 2011]



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2h$ (c) $2\pi a^2h$ (d) Zero

33. Identify the CORRECT statement for the following vectors $\vec{a}=3\hat{i}+2\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}$.

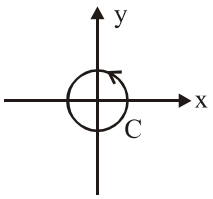
(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent [GATE 2012]

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

34. Given $\vec{F}-\vec{r}\times\vec{B}$, where $\vec{B}=B_0\left(\hat{i}+\hat{j}+k\right)$ is a constant vector and r is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is :



[GATE 2012]

(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

35. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\vec{\nabla}\left[\vec{A}\left(\vec{B}\times\vec{r}\right)\right]$ is : [GATE 2013]

QUANTA

(a) $\vec{A}.\vec{B}$ (b) $\vec{A}\times\vec{B}$ (c) $\vec{r}$ (d) zero

36. The unit vcctor perpendicular to the surface $x^2+y^2+z^2=3$ at the point (1, 1, 1) is [GATE 2014]

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

37. Four forces arc Riven below in Cartesian and spherical polar coordinates. [GATE 2015]

(i) $\vec{F}_1=K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (b) $\vec{F}_2=K\left(x^3\hat{y}-y^3\hat{z}\right)$

(c) $\vec{F}_3=K\left(x^3\hat{x}+y^3\hat{y}\right)$ (d) $\vec{F}_4=K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.

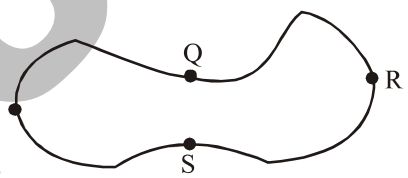
(a) (iii) and (iv) are conservative but (i) and (ii) are not

(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

38. Given that magnetic flux through the closed loop PQRSP is ϕ. If $\int_P^R \vec{A}\cdot\vec{dl}=\phi_1$ along PQR, the value of $\int_P^R \vec{A}\cdot\vec{dl}$ along PSR is [GATE 2015]



(a) $\phi-\phi_1$ (b) $\phi_1-\phi$ (c) $-\phi_1$ (d) ϕ_1

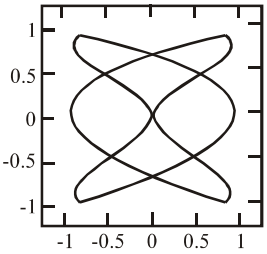
39. The direction of $\vec{\nabla}f$ for a scalar field $f(x,y,z)=\frac{1}{2}x^2-xy+\frac{1}{2}z^2$ at the point P(1, 1, 2) is : [GATE 2016]

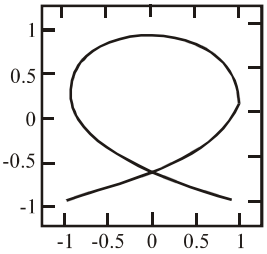
(a) $\frac{\left(-\hat{j}-2\hat{k}\right)}{\sqrt{5}}$ (b) $\frac{\left(-\hat{j}+2\hat{k}\right)}{\sqrt{5}}$ (c) $\frac{\left(\hat{j}-2\hat{k}\right)}{\sqrt{5}}$ (d) $\frac{\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}}$

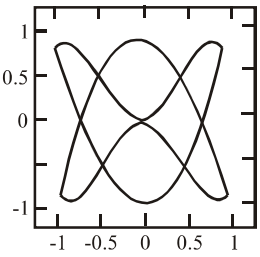
40. A two-dimensional vector $\vec{A}(t)$ is given by $\vec{A}(t)=\hat{i}\sin 2t+\hat{j}\cos 3t$

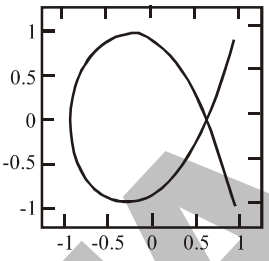
Which of the following graphs besecribes the locus of the tip of the vector, as t is varied from 0 to 2π ? [TIFR 2013]

QUANTA

(a) 

(b) 

(c) 

(d) 

41. Consider the surface corresponding to the equation $4x^2 + y^2 + z = 0$. A possible unit tangent to this surface at the point (1, 2, –8) is : [TIFR 2013]

(a) $\frac{1}{\sqrt{2}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$

(b) $\frac{1}{5}\hat{j} - \frac{4}{5}\hat{k}$

(c) $\frac{4}{9}\hat{i} - \frac{8}{9}\hat{j} + \frac{1}{9}\hat{k}$

(d) $-\frac{1}{\sqrt{5}}\hat{i} + \frac{3}{\sqrt{5}}\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$

42. Which of the following vectors is parallel to the surface $x^2y + 2xz = 4$ at the point (2, –2, 3)? [TIFR 2015]

(a) $-6\hat{i} - 2\hat{j} + 5\hat{k}$ (b) $6\hat{i} + 2\hat{j} + 5\hat{k}$ (c) $6\hat{i} - 2\hat{j} + 5\hat{k}$ (d) $6\hat{i} - 2\hat{j} - 5\hat{k}$

43. The vector field $xz\hat{i} + y\hat{j}$ in cylinclerical polar coordinates is : [JEST 2013]

(a) $\rho\left(z\cos^2\phi + \sin^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1-z)\hat{e}_\phi$

(b) $\rho\left(z\cos^2\phi + \sin^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1+z)\hat{e}_\phi$

(c) $\rho\left(z\sin^2\phi + \cos^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1+z)\hat{e}_\phi$

(d) $\rho\left(z\sin^2\phi + \cos^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1-z)\hat{e}_\phi$

44. Given that the coordinates of particle are $y(t) = A\cos(2\omega t)$ and $x(t) = \sin(\omega t)$ the trajectory of the particle is a :

(a) Circle (b) Ellipse (c) Hyperbola (d) Parabola

45. A point particle is moving in the (x, y) plane on a trajectory given in polar coordinates by the equation $r\sin\left(\theta + \frac{\pi}{4}\right) = 5$. The trajectory of the particle is :

QUANTA

(a) a parabola (b) a straight line (c) a circle (d) a hyperbola

46. A point particle is moving the (x, y) plane on a trajectory given in polar coordinates by the equation

$$r^2 - 2r\sin\left(\theta + \frac{\pi}{4}\right) - 3 = 0$$

The trajectory of the particle is :

(a) a parabola (b) a straight line (c) a circle (d) a hyperbola

47. The normal to the surface given by the equation $z = \cos x \cosh y$ at the point $x = \frac{\pi}{2}$ and $y = 0$ lies in

(a) (x, y) plane (b) (x, z) plane

(c) (y, z) plane (d) On the plane given by the equation $x + y + z = \frac{\pi}{2} + 1$

48. A particle is moved quasi-statically from the point (–3, 0) to (3, 0), along a path $y = x^2 - 9$ in an external force field given by $\vec{F} = yi + 3yj$. Give that all physical quantities are in SI units, the magnitude of the work done on the particle is given by :

(a) 36 J (b) 18 J (c) 9 J (d) 0

49. A point particle is moving in the (x, y) plane on a trajectory given in polar coordinates by the equation

$$25 + r^2\cos 2\theta = 0$$

The trajectory of the particle is a :

(a) parabola (b) circle (c) ellipse (d) hyperbola

50. The value of the line integral

$$\oint \frac{xdy - ydx}{x^2 + y^2}$$

along a circle of radius 3 centered at the origin tri the counter clockwise direction is given by :

(a) 0 (b) $\frac{3}{2\pi}$ (c) 2π (d) 6π

51. $\nabla^2\left(\frac{1}{r}\right)$ is :

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$

52. $\iiint \nabla^2\left(\frac{1}{r}\right)dV$ $r \neq 0$ is :

QUANTA

(a) 0 (b) -4π (c) 4π (d) 1

53. The value oft for which three vectors :

$\left[(1-t),0,0\right],\left[1,(1-t),0\right]\&\left[1,1,(1-t)\right]$ are linearly dependent is :

(a) 1 (b) 0 (c) 2 (d) –1

54. The value of $\oint \vec{A}.\overrightarrow{d\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is :

(a) 0 (b) 2LA (c) 4LA (d) L^2A

55. The value of $\oint_S \frac{\vec{r}\cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is :

(a) 0 (b) π (c) 4π (d) 8π

56. If $\vec{A}(t)$ is a vector o f constant magnitude, which of the following is true?

(a) $\frac{d\vec{A}}{dt}=0$ (b) $\frac{d^2\vec{A}}{dt^2}=0$ (c) $\frac{d\vec{A}}{dt}\bullet\vec{A}=0$ (d) $\frac{d\vec{A}}{dt}\times\vec{A}=0$

57. The function $r^n\hat{r}(r>0)$, where r refers to spherical coordinate system, is:

(a) an irrotationat vector for $n=-2$ (b) a solenoidal vector for $n=-2$

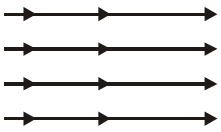
(c) an irrotational vector for $n=-3$ (d) a solenoidal vector for $n=-3$

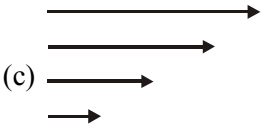
58. Two sets of vectors, one with m elements and the other with n elements (m < n) span the same linear vector space. If k is the dimension of the vector space, then :

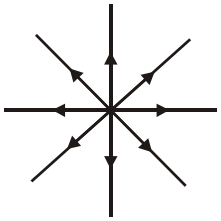
(a) $k\le m$ (b) $k\ge m$ (c) $k=m+n$ (d) $k=n$

59. Identify the vector field given below which has a finite curl :

(a)

(b) 

(c) 

(d) 

60. Given any three non-zero vectors $\vec{A},\vec{B}$ and $\vec{C}$, their triple product $\vec{A}.\left(\vec{B}\times\vec{C}\right)$ vanishes if

(a) They are perpendicular to each other (b) Any two of them are perpendicular

(c) Any twn of them are parallel (d) They are non-coplanar

QUANTA

61. The necessary and sufficient condition that $\oint_c \vec{A}.d\vec{r} = 0$, for an v closed curve C is

(a) $\vec{\nabla}.\vec{A} = 0$ (b) $\vec{\nabla} \times \vec{A} = 0$ (c) $\vec{\nabla}.\vec{\nabla} \times \vec{A} = 0$ (d) $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = 0$

62. Any arbitrary vector in a three dimnensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors :

(a) Arbitrary number (b) 1 (c) 2 (d) 3

63. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to :

(a) Curl of a vector function (b) Gradient of a vector function

(c) Gradient of a vector function (d) Zero

64. When the fluid is incompressible, the equal of continuity can be reduced to $\frac{\partial \rho}{\partial t} = 0 = \rho\left(\vec{\nabla}.\vec{v}\right)$. Since the density ρ is constant in this case. Here $\vec{v}$ is the velocity of a typical particle of the fluid. Further, if the flow is irrotational, then the equation can be rewritten as :

(a) $\vec{\nabla}.\vec{\nabla}\phi = 0$ (b) $\vec{\nabla} \times \vec{\nabla}\phi = 0$ (c) $\vec{\nabla}\phi = 0$ (d) $\vec{v} =$ constant

65. Given the vector $\vec{A}(y, -x, 0)$, the line integral $\oint_c \vec{A}.\vec{dl}$, where C is a circle of radius 5 units with itscentre at the origin, (correct to the first decimal place) is :

(a) 172.8 (b) 157.1 (c) –146.3 (d) 62.8

66. The value of integral $1 = \int_s \vec{r}.d\vec{s}$ where S is the surface enclosing volume V is :

(a) 3 (b) V (c) 3V (d) 0

ANSWER-KEY

1.(a)	2.(b)	3.(c)	4.(a)	5.(c)	6.(d)	7.(d)
8.(d)	9.(d)	10.(b)	11.(c)	12.(c)	13.(a)	14.(c)
15.(d)	16.(d)	17.(a)	18.(a)	19.(d)	20.(a)	21.(d)
22.(d)	23.(a)	24.(c)	25.(a)	26.(b)	27.(a)	28.(c)
29.(b)	30.(d)	31.(b)	32.(b)	33.(a)	34.(c)	35.(b)
36.(d)	37.(d)	38.(b)	39.(b)	40.(c)	41.(a)	42.(c)
43.(a)	44.(d)	45.(b)	46.(c)	47.(b)	48.(a)	49.(d)
50.(c)	51.(c)	52.(a)	53.(a)	54.(a)	55.(c)	56.(c)
57.(b)	58.(b)	59.(c)	60.(c)	61.(b)	62.(d)	63.(b)
64.(b)	65.(b)	66.(c)				

QUANTA

SECTION: VECTOR ALGEBRA

Consider the matrix $M=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$ [CSIR June 2011]

1. The eigenvalues of M are

(a) 0,1,2 (b) 0, 0, 1 (c) 1, 1, 1 (d) 1, 1, 3

2. The exponential of Msimplifies (I is the-3 × 3 identity matrix) :

(a) $e^{M}=1+\left(\frac{e^{3}-1}{3}\right)M$ (b) $e^{M}=1+M+\frac{M^{2}}{2!}$

(c) $e^{M}=1+3^{3}M$ (d) $e^{M}=(e-1)M$

3. A 3 × 3 matrix M has $Tr(M)=6, Tr\left[M^{2}\right]=26, Tr\left[M^{3}\right]=90$. Which of the following can be possible set of eigenvalues of M? [CSIR June 2011]

(a) {1, 1, 4} (b) {–1, 0, 7} (c) {–1, 3, 4} (d) {2, 2, 2}

4. The eigenvalues of the matrix $A=\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{pmatrix}$ [CSIR June 2012]

(a) (1, 4, 9) (b) (0, 7, 7) (c) (0, 1, 13) (d) (0, 0, 14)

5. The eiegenvalues of the anti-symmetric matrix $A=\begin{pmatrix}0 & -n_3 & n_2\\ n_3 & 0 & -n_1\\ -n_2 & n_1 & 0\end{pmatrix}$, where n_1, n_2, n_3 are components of an unit vector, are [CSIR June 2012]

(a) 0, –i, i (b) 0, 1, –1 (c) 0, 1 +i, (d) 0, 0, 0

6. A 2 × 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^{n}=I$: [CSIR June 2012]

(a) 20 (b) 30 (c) 60 (d) 120

7. Given a 2 × 2 unitary matrix satisfying $U'U=UU'=1$ with $\det U=e^{i\varphi}$, one can construct a unitary matrix $V\left(V'V=VV'=1\right)$ with $\det V=1$ from it by [CSIR Dec. 2012]

(a) Multiplying U by $e^{-i\varphi/2}$

(b) Multiplying a single element of U by $e^{-i\varphi}$

(c) Multiplying any row or column by $e^{-i\varphi/2}$

(d) Multiplying U by $e^{-i\varphi}$

QUANTA

8. Consider an $n \times n (n > 1)$ matrix A, in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$). The matrix A : [CSIR Dec. 2013]

(a) has one degenerate eigenvalue with degeneracy $(n-1)$

(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$

(c) has one degenerate eigenvalues with degeneracy n

(d) does no t have any degenerate eigenvalues

9. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are [CSIR June 2014]

(a) –5, –2, 7 (b) –7, 0, 7 (c) –4i, 2i, 2i (d) 2, 3, 6

10. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relation : [CSIR June 2014]

(a) $[A,B] = B + C, [B,C] = 0, [C,A] = B + C$ (b) $[A,B] = C, [B,C] = A, [C,A] = B$

(c) $[A,B] = B, [B,C] = 0, [C,A] = A$ (d) $[A,B] = C, [B,C] = 0, [C,A] = B$

11. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvcctor of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if :

(a) $b = 0$ and $a = 0$ (b) $b = a$ and $b = -2a$

(c) $b = 2a$ and $b = -a$ (d) $b = a/2$ and $b = -a/2$

12. The matrix $M = \begin{pmatrix} 1 & 3 & 2 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ satisfies the equation : [CSIR Dec. 2016]

(a) $M^3 - M^2 - 10M + 12I = 0$ (b) $M^3 - M^2 - 12M + 10I = 0$

QUANTA

(c) $M^3 - M^2 - 10M + 10I = 0$ (d) $M^3 + M^2 - 10M + 10I = 0$

13. For any of)erator $A, i\left(A^+ - A\right)$ is : [GATE 2001]

(a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal

14. If two matrices A and B can be diagonalized simultaneously, which of the following is true? [GATE 2002]

(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$

(c) $AB = BA$ (d) $AB^2AB = BABA^2$

15. Which one of the following matrices is the inverse ofthe matrix $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$ [GATE 2002]

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

16. A 3 × 3 matrix has eigenvalues 0, 2 +i and 2 –i. Which one of following statements is correct? [GATE 2003]

(a) The matrix is Hermitian (b) The matrix is unitary

(c) The inverse of the matrix (d) The determinant of the matrix is zero

17. A real traceless 4 × 4 unitary matrix is has two eigen values –1 and 1. The other eigenvalues are : [GATE 2004]

(a) zero and +2 (b) –1 and +1 (c) zero and +1 (d) +1 and +1

18. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are : [GATE 2004]

(a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) –1 and +1

19. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are : [GATE 2005]

(a) –2, –1, 1, 2 (b) –1, 1, 0, 2 (c) 1, 0, 2, 3 (d) –1, 1, 0, 3

20. The determinant of a 3 × 3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvalue is : [GATE 2005]

(a) 4 (b) 6 (c) 8 (d) 9

21. A linear transformation T, de6ned as $T = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transforrn a vector $\vec{x}$ from a three-dimensional real space to a two-dimensional real space. The transformation matrix T is: [GATE 2006]

QUANTA

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Common data for Q. 22 and Q. 23 :

One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5. [GATE 2006]

22. The other two eigen values are:

(a) 0 and 0 (b) 1 and 1 (c) 1 and –1 (d) –1 and –1

23. The normalized eigen vector corresponding to the eigen value 5 is:

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

24. The eigenvalues of a matrix are i,–2i and 3i. The matrix is : [GATE 2007]

(a) unitary (b) anti-unitary (c) hermitian (d) anti-hermitian

25. The eigenvalues and eigenvectois:of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are : [GATE 2007]

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

26. For arbitary matrices E, F, G and H, if $EF - FE = 0$, then Trace (*EFGH*) is equal to [GATE 2008]

(a) Trace (HGFE) (b)Trace (E).Trace(F).Trace (G).Trace (H)

(c) Trace (GFEH) (d) Trace (EGHF)

27. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is : [GATE 2008]

(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$ (b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$ (c) $\begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}$ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$

28. The eigenvalues of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ are : [GATE 2008]

QUANTA

(a) $\frac{1}{2}\left(\sqrt{3}\pm i\right)$ when $\theta = 45°$ (b) $\frac{1}{2}\left(\sqrt{3}\pm i\right)$ when $\theta = 30°$

(c) ± 1 since, the matrix is unitary (d) $\frac{1}{2}\left(1\pm i\right)$ when $\theta = 30°$

29. The eigenvaluess ofthe matrix $A=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$ are : [GATE 2009]

(a) real and distinct (b) complex and distinct

(c) complex and coinciding (d) real and coinciding

30. The eigen values olthe matrix $\begin{pmatrix}2 & 3 & 0\\ 3 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}$ are : [GATE 2010]

(a) 5, 2, –2 (b) –5, 1, –1 (c) 5, 1, –1 (d) –5, 1, 1

31. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which ofthe following statements is NOT TRUE? [GATE 2011]

(a) Det A = Det B (b) Trate of A = Trice of B

(c) A and B have the same eigenyectors (d) A and B have the same eigenvalues

32. A 3 × 3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is : [GATE 2011]

(a) 18 (b) 12 (c) 9 (d) 6

33. The eigenvalues of the matrix $\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$ are : [GATE 2012]

(a) 0, 1, 1 (b) $0,-\sqrt{2},\sqrt{2}$ (c) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,$ (d) $\sqrt{2},-\sqrt{2},0$

34. The degenerate eigenvalues of the matrix $M=\begin{pmatrix}4 & -1 & -1\\ -1 & 4 & -1\\ -1 & -1 & 4\end{pmatrix}$ is :

(your answer should be an integer) [GATE 2013]

35. The matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\ 1-i & -1\end{bmatrix}$ is : [GATE 2012]

(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

36. The matrix $\begin{pmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 1\end{pmatrix}$ [GATE 2010]

QUANTA

can be relateed by a similarity transformation to the matrix.

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

37. Consider the matrix, $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$

A 3-dimensional basis formed by eigenvectors of M is : [TIFR 2011]

(a) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

38. This trace of the real 4×4 matrix $U = \exp(A)$, where $A = \begin{pmatrix} 0 & 0 & 0 & \pi/4 \\ 0 & 0 & \pi/4 & 0 \\ 0 & \pi/4 & 0 & 0 \\ \pi/4 & 0 & 0 & 0 \end{pmatrix}$ is

equal to : [TIFR 2011]

(a) $2\sqrt{2}$ (b) $\pi/4$ (c) $\exp(i\varphi)$ for $\varphi = 0, \pi$ (d) zero

39. Two different 2×2 matrices A and B are found to have the same eigenvalues. It is then correct to state that $A = SBS^{-1}$ where S can be a : [TIFR 2012]

(a) traceless 2×2 matrix (b) Hermitian 2×2 matrix

(c) unitary 2×2 matrix (d) arbitrary 2×2 matrix

40. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that : [TIFR 2012]

(a) $\{M, N\} = 0$ (b) $[M, N] = 0$ (c) $M^\dagger = N$ (d) $M^\dagger = N^{-1}$

41. If the eigenvalues of a symmetric 3×3 matrix A are 0, 1, 3 and the corresponding eigenvectors can be written as : [TIFR 2016]

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

respectively, then the matrix A^4 is

QUANTA

(a) $\begin{bmatrix} 41 & -81 & 40 \\ -81 & 0 & -81 \\ 40 & -81 & 41 \end{bmatrix}$ (b) $\begin{bmatrix} -82 & -81 & 79 \\ -81 & 81 & -81 \\ 79 & -81 & 83 \end{bmatrix}$

(c) $\begin{bmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{bmatrix}$ (d) $\begin{bmatrix} 14 & -13 & 27 \\ -13 & 54 & -13 \\ 27 & -13 & 14 \end{bmatrix}$

42. Denote the commutator of two matrices A and B by $[A,B]=AB-BA$ and the anti-commutator by $\{A,B\}=AB+BA$. If $\{A,B\}=0$, we can write $[A,BC]=$ [TIFR 2017]

(a) $B[A,C]$ (b) $B(A,C)$ (c) $-B(A,C)$ (d) $[A,C]B$

43. The matrix $\begin{pmatrix} 100\sqrt{2} & x & 0 \\ -x & 0 & -x \\ 0 & x & 100\sqrt{2} \end{pmatrix}$, where x > 0, is known to have-two equal eigenvalues. Find the valit of x. [TIFR 2017]

44. A unitary matrix U is expanded in terms of a Hermitian matrix H, such that $U=e^{i\pi H/2}$. If we know that :

$$H=\begin{pmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

then U must be : [TIFR 2017]

(a) $\begin{pmatrix} i & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & i & \frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & i \end{pmatrix}$ (b) $\begin{pmatrix} \frac{i}{2} & 0 & \frac{+\sqrt{3}}{2} \\ 0 & i & 0 \\ \frac{i\sqrt{3}}{2} & 0 & -\frac{i}{2} \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ \sqrt{3} & 0 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2i & 1 & \frac{\sqrt{3}}{2} \\ 1 & 2i & 0 \\ \frac{\sqrt{3}}{2} & 0 & 2i \end{pmatrix}$

45. For an $N \times N$ matrix consisting of all ones,

(a) all eigenvalues = 1 (b) all eigenvalues = 0

(c) the eigenvalues are 1, 2, ..., N (d) one eigenvalue = N, the others = 0

46. The coordinate transformation [JEST 2013

$x'=0.8x+0.6y$, $y'=0.6x-0.8y$

represents

(a) a translation (b) a proper rotation (c) a reflection (d) none of the above

QUANTA

47. Given a matrix $M=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$, which of the following represents $\cos(\pi M/6)$.[JEST 2016]

(a) $\frac{1}{2}\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ (c) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}$

48. The eigenvalues of the matrix $\begin{bmatrix}0 & 16\\ 16 & 0\end{bmatrix}$ are :

(a) 1 and –1 (b) 16 and 16 (c) 16 and –16 (d) 1 and 256

49. Consider the matrix, $A=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\end{pmatrix}$

the eigenvalues of A are, $\left(i=\sqrt{-1}\right)$

(a) $\{1,-i,-1,i\}$ (b) $\{1,-1,0,0\}$ (c) $\{1,1,1,1\}$ (d) $\{1,1,i,-i\}$

50. Let P be a n × n diagonalizable matrix which satisfies the equations :

$$P^2=P, Tr(P)=n-1$$

Det (P) is :

(a) n (b) 0 (c) 1 (d) n–1

51. Let M be a 3 × 3 Hermitian matrix which satisfies the matrix equation :

$$M^2=5M+6I=0$$

Where I refers to the identity matrix. Which of the following are possible eigenvalues of M

(a) {1, 2, 3} (b) {2, 2, 3} (c) {2, 3, 5} (d) {5, 5, 6}

52. Given the three matrices

$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ and $\left[\sigma_i,\sigma_j\right]=\sigma_i\sigma_j-\sigma$

then $\left[\sigma_1,\left[\sigma_2,\sigma_3\right]\right]+\left[\sigma_2,\left[\sigma_3,\sigma_1\right]\right]+\left[\sigma_3,\left[\sigma_1,\sigma_2\right]\right]$ is :

(a) $\sigma_1^2+\sigma_2^2+\sigma_3^2$ (b) $\sigma_1+\sigma_2+\sigma_3$ (c) 0 (d) Identity

53. Given the three matrices

$$\sigma_i=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \sigma_2=\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}, \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

QUANTA

Which of the following statements is true for all positive integers n and i = 1, 2, 3 ?

(a) $\sigma_i^n = I$ (b) $\sigma_i^n = \sigma_i$ (c) $\sigma_i^{2n} = I$ (d) $\sigma_i^{2n} = \sigma_i$

54. The trace of a 2 × 2 matrix is 1 and its determinate is 1. Which ofthe following has to be true ?

(a) One of the eigenvalues is 0 (b) One of the eigenvalue is 1

(c) Both of the eigenvalues are 1 (d) Neither of the eigenvalues is 1

55. Let M by a 3 × 3 Hermitain matrix which satisfies the matrix equation :

$$M^2 - 7M + 12I = 0$$

where I refers to the identity matrix. What is the determinant of the matrix M given that the trace is 10 ?

(a) 27 (b) 36 (c) 48 (d) 64

56. Consider a 3 × 3 matrix of the form :

$$\begin{pmatrix} a_1^2 & a_1a_2 & a_1a_1 \\ a_1a_2 & a_2^2 & a_2a_3 \\ a_1a_3 & a_2a_3 & a_3^2 \end{pmatrix}$$

The number of zero eigenvalues for this matrix is :

(a) 0 (b) 1 (c) 2 (d) 3

57. Which one of the following is not hermitian ?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$

58. If A is an antisymmetric matrix and A^T be the transpose of this matrix, then which one of the following relations does not hold good?

(a) $A^T = A$ (b) $AA^T = A^TA$

(c) A^2 is an antisymmetric matrix (d) A^2 is a syrrgnetire matrix

59. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are given by λ_1, λ_2 and λ_3. Which one of the following statement is NOT TRUE?

(a) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (b) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$

(c)All eigenvalues are real (d) Product of the eigenvalues is 1

60. The matrix $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is :

QUANTA

(a) orthogonal (b) hermitian (c) symmetric (d) anti-symmetric

61. A given (n × n) nilpotent matrix A satisfies the equation $A^k = 0$ for $1 < k < n$. Therefore

(a) Exactly k eigenvalues of A must be zero

(b) Exactly (n – k) eigenvalue-of A must be zero

(c) Every eigenvalue of A is zero

(d) A can have (n – 1) non-zero eigenvalues

62. If L(x) is a linear differential operator and $y_1(x), y_2(x)$ are two arbitrary functions. Then

(a) $L(x)(y_1 + y_2) = Ly_1 + Ly_2$ (b) $L(x)(y_1 + y_2) = y_1Ly_2 + y_2Ly_1$

(c) $L(x)(y_1y_2) = Ly_1 + Ly_2$ (d) $L(x)(y_1 - y_2) = Ly_1 + Ly_2$

63. For which of the following values of a, the inverse of the matrix, $M = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 3 & 5 \\ 0 & 4 & a \end{pmatrix}$, does not exist?

(a) 4 (b) 4 (c) 0 (d) 1

64. Which one of the following matrices is orthogonal?

(a) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$

65. The number of independent real parameters of a most general hermit ian matrix of order 4 is:

(a) 4 (b) 8 (c) 16 (d) 32

66. The eigen values of the matrix $\begin{bmatrix} 0 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & 0 \end{bmatrix}$ are :

(a) Purely imaginary (b) Complex (c) Real (d) Zero

67. If a matrix A satisfies the condition $A^2 = I$ and $Tr(A) = 0$ (where I is a n × n identity matrix), then

(a) The determinant of A must be 0 (b) The determinant of A must be +1

(c) A must be odd dimensional (d) A must be even dimensional

68. The matrix A, defined by $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$ is orthogonal if :

QUANTA

(a) $a=1, b=-1$ (b) $a=1/\sqrt{2}, b=-1/\sqrt{2}$

(c) $a=1/\sqrt{2}, b=-i/\sqrt{2}$ (d) $a=1, b=1$

69. If A and B are two n × n matrices, then the trace of $C=[A,B]$ is :

(a) $Tr(AB)$ (b) $Tr(A)Tr(B)$ (c) 0 (d) $Tr(A)+Tr(B)$

70. Consider the matrix $M=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a and c are real and b may be complex. If M can be diagonalized by a matrix S, then S is :

(a) diagonal (b) symmetric (c) orthogonal (d) unitary

71. The trace and the determinant of a 2×2 matrix are denoted by T and D. The eigenvalues of the matrix are given by :

(a) $\frac{T \pm \sqrt{T^2-4D}}{2}$ (b) $\pm\frac{\sqrt{T^2-4D}}{2}$ (c) $\frac{T \pm D}{2}$ (d) $\pm\frac{T}{2}$

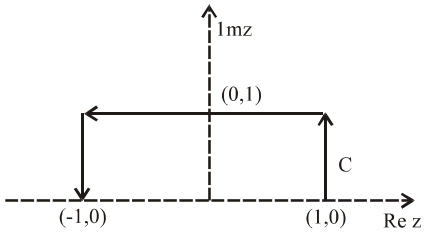
ANSWER-KEY

1.(b)	2.(a)	3.(c)	4.(d)	5.(a)	6.(c)	7.(a)
8.(a)	9.(b)	10.(d)	11.(b)	12.(c)	13.(a)	14.(c)
15.(c)	16.(d)	17.(b)	18.(c)	19.(a)	20.(b)	21.(a)
22.(c)	23.(d)	24.(d)	25.(a)	26.(a)	27.(d)	28.(b)
29.(b)	30.(c)	31.(c)	32.(d)	33.(b)	34.(5)	35.(d)
36.(d)	37.(b)	38.(a)	39.(c)	40.(a)	41.(c)	42.(c)
43.()	44.(b)	45.(d)	46.(c)	47.(b)	48.(c)	49.(a)
50.(b)	51.(b)	52.(c)	53.(c)	54.(d)	55.(b)	56.(c)
57.(d)	58.(c)	59.(c)	60.(a)	61.(c)	62.(a)	63.(b)
64.(b)	65.(a)	66.(c)	67.(d)	68.(b)	69.(c)	70.(d)
71.(a)						

QUANTA

SECTION : COMPLEX ANALYSIS

1. The value of the integral $\int_C dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in figure below: [CSIR June 2011]



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$ (c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

2. Which of the following is an analytic function of the complex variable $z=x+iy$ in the domain $|z|<2$? [CSIR June 2011]

(a) $(3+x-iy)^7$ (b) $(1+x+iy)^4(7-x-iy)^3$

(c) $(1-2x-iy)^4(3-x-iy)^3$ (d) $(x+iy-1)^{1/2}$

3. The first few terms in the Taylor series expansion of the function $f(x)=\sin x$ around $x=\frac{\pi}{4}$ are : [CSIR Dec. 2011]

(a) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)+\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+.....\right]$

(b) $\frac{1}{\sqrt{2}}\left[1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+.....\right]$

(c) $\left[\left(x-\frac{\pi}{4}\right)-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+.....\right]$

(d) $\frac{1}{\sqrt{2}}\left[1-x\frac{x^2}{2!}-\frac{x^3}{3!}+.....\right]$

4. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1\le|z|\le 2$ and around $z=1$ is : [CSIR Dec. 2011]

(a) $\frac{1}{2}\left[1+z+z^2+...\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+...\right]$ (b) $\frac{1}{1-z}-z-(1-z)^2+(1-z)^3+...$

QUANTA

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+\ldots\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+\ldots\right]$ (d) $2(1-z)+5(1-z)^2+7(1-z)^3+\ldots$

5. The value of the integral $\int_{-\infty}^{\infty}\frac{1}{t^2-R^2}\cos\left(\frac{\pi t}{2R}\right)dt$ [CSIR June 2012]

(a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

6. Let $u(x,y)=x+\frac{1}{2}\left(x^2-y^2\right)$ be the real part of an analytic function $f(z)$ of the complex variable $z=x+iy$ the imaginary part of $f(z)$ is : [CSIR June 2012]

(a) $y\,|\,xy$ (b) xy (c) y (d) y^2-x^2

7. The taylor series expansion of the function $\ln(\cosh x)$, where x is real, about point x = 0 starts with the following terms : [CSIR Dec. 2012]

(a) $-\frac{1}{2}x^2+\frac{1}{12}x^4+\ldots$ (b) $\frac{1}{2}x^2-\frac{1}{12}x^4+\ldots$ (c) $-\frac{1}{2}x^2+\frac{1}{6}x^4+\ldots$ (d) $\frac{1}{2}x^2+\frac{1}{6}x^4+\ldots$

8. The value of the integral $\oint_C\frac{z^2}{z^2-5z+6}dz$, where C is closed contour defined by the equation $2|z|-5=0$, traversed in the anti-clockwise direction is, [CSIR Dec. 2012]

(a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

9. With $z=x+iy$, which of the following functions $f(x,y)$ is NOT a complex analytic funetion of z? [CSIR June 2013]

(a) $(x+iy-8)^3\left(4+x^2-y^2+2ixy\right)^7$ (b) $(x+iy)^7(1-x-iy)^3$

(c) $\left(x^2-y^2+2ixy-3\right)^5$ (d) $(1-x+iy)^4(2+x+iy)^6$

10. Which of the following functions cannot be the real part of a complex analytic function of $z-x+iy$? [CSIR Dee 2013]

(a) x^2y (b) x^2-y^2 (c) x^2-3xy^2 (d) $3x^2y-y-y^3$

11. Given that the integral $\int_0^{\infty}\frac{dx}{y^2+x^2}=\frac{\pi}{2y}$, the value of $\int_0^{\infty}\frac{dx}{\left(y^2+x^2\right)}$ is : [CSIR Dee 2013]

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

QUANTA

12. If C is the contour defined by $|z|=\frac{1}{2}$, the value of the integral $\oint_C \frac{dz}{\sin^2 z}$ is :

[CSIR Dee 2013]

(a) ∞ (b) $2\pi i$ (c) 0 (d) πi

13. The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3} dx$ is : [CSIR June 2014]

(a) -2π (b) $-\pi$ (c) π (d) 2π

14. The Laurent series expansion of the function $f(z)=e^z+e^{1/\pi}$ about z = 0 is :

[CSIR June 2014]

(a) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ for all $|z|<\infty$ (b) $\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ only if $0<|z|<1$

(c) $\sum_{n=0}^{\infty}\left(z^n+\frac{1}{z^n}\right)\frac{1}{n!}$ for all $0<|z|<\infty$ (d) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ only if $|z|<1$

15. Consider the function $f(z)=\frac{1}{2}\ln(1-z)$ of a complex variable

$z=re^{i\theta}\left(r\geq 0,-\infty<\theta<\infty\right)$. The singularities of $f(z)$ are as follows:[CSIR Dee. 2014]

(a) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ only for $0\leq\theta<2\pi$

(b) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ only for θ, other than $0\leq\theta<2\pi$

(c) branch points at $z=1$ and $z=\infty$; and a pole at $z=0$ only for θ

(d) branch points at $z=0, z=1$ and $z=\infty$

16. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx$ is : [CSIR June 2015]

(a) $\pi/\sqrt{2}$ (b) $\pi/2$ (c) $\sqrt{2}\pi$ (d) 2π

17. The function $\frac{z}{\sin \pi z^2}$, of a complex variable z has : [CSIR Dec. 2015]

(a) a simple pole at 0 and poles of order 2 at $z=\pm\sqrt{n}$ for n = 1, 2, 3...

(b) a simple pole at 0 and poles of order 2 at $z=\pm\sqrt{n}$ and $z=\pm i\sqrt{n}$ for n = 1, 2, 3....

(c) poles of order 2 at $z=\pm\sqrt{n}$ for n 0, 1, 2, 3....

(d) poles of order 2 at $z=\pm\sqrt{n}$ for n = 0, 1, 2, 3....

18. The radius of convergence of the Taylor series expansion of the function $\frac{1}{\cosh(x)}$ around x = 0, is : [CSIR June 2015]

QUANTA

(a) ∞ (b) π (c) $\pi/2$ (d) 1

19. The value of the contour integral $\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$ around the unit circle C traversed in the anti-clockwise direction, is : [CSIR June 2016]

(a) 0 (b) 2 (c) $-\frac{8}{\sqrt{3}}$ (d) $-\tanh\left(\frac{1}{2}\right)$

20. The value of the integral $\int_C z^{10}dz$, where C is the unit circle with the origin as the centre is: [GATE 2001]

(a) 0 (b) $z^{11}/11$ (c) $2\pi i z^{11}/11$ (d) 1/11

21. The value of the residue of $\frac{\sin z}{z^6}$ is : [GATE 2001]

(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$ (c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

22. If a function $f(z)=u(x,y)+iv(x,y)$ of the complex variable $z=x+iy$, where x,y,u and v are real, is analytic in a domain D of z, then which of the following is true? [GATE 2002]

(a) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 v}{\partial x\partial y}$

23. The value of the integral $\int_C dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is: [GATE 2003]

(a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$

24. The inverse of the complex number $\frac{3+4i}{3-4i}$ is : [GATE 2004]

(a) $\frac{7}{25}+i\frac{24}{25}$ (b) $-\frac{7}{25}+i\frac{24}{25}$ (c) $\frac{7}{25}-i\frac{24}{25}$ (d) $-\frac{7}{25}-i\frac{24}{25}$

25. The value of $\int_C \frac{dz}{\left(z^2+a^2\right)}$, wherc C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is :

QUANTA

(a) π for $a=2$ (b) zero for $a=\frac{1}{2}$ (c) 4π for $a=2$ (d) $\frac{\pi}{2}$ and $a=\frac{1}{2}$

26. The value of the integral $\int_C \frac{dz}{z+3}$, where C is a circle (anticlockwise) with $|z|=4$ is :

(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$ [GATE 05]

27. All solutions of the equation $e^z=-3$ are :

(a) $in\pi \ln 3, n=\pm1,\pm2.....$ (b) $\ln 3+i(2n+1)\pi, n=0,\pm1,\pm2.....$

(c) $\ln 3+i2n\pi, n=0,\pm1,\pm2.....$ (d) $i3n\pi, n=\pm1,\pm2.....$

28. The value of $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is a circle defined by $|z|=3$ is: [GATE 2006]

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$ (c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^{2}$

29. The contour integral $\oint \frac{dz}{z^2+a^2}$, is to be valuated on a circle of radius 2a centered at the origin. It will have contributions only from the points. [GATE 2007]

(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (b) ia and $-ia$

(c) $ia,-ia,\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (d) $\frac{1+i}{\sqrt{2}}a-\frac{1+i}{\sqrt{2}}a,\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$

30. If $I=\oint_C \ln z\,dz$, where C is the unit circle taken anticlockwise and ln z is the principal branch ofthe logarithmic function, which of the following is correct? [GATE 2008]

(a) $I=0$ by residue theorem

(b) I is not defined since, ln z is branch cut

(c) $I\neq 0$

(d) $\oint_C \ln\left(z^2\right)dz=2I$

3l. The value of $\int_{-i}^{i}\pi(z+1)dz$ is [GATE 2008]

(a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) $(-1+2i)\pi$

32. The value of the integral $\int_C \frac{e^z}{z^2-3z+2}dz$, where the contour C is the circle $|z|=\frac{3}{2}$ is :

QUANTA

[GATE 2009]

(a) $2\pi ie$ (b) πie (c) $-2\pi ie$ (d) $-\pi ie$

33. The value of the integral $\oint_C \frac{e^z \sin z}{z^2} dz$, where the contour C is the unit circle: $|z-2|=1$, is : [GATE 2010]

(a) $2\pi i$ (b) $4\pi i$ (c) πi (d) 0

34. For the complex function, $f(z)=\frac{e^{\sqrt{2}}-e^{-\sqrt{2}}}{\sin\left(\sqrt{z}\right)}$, which of the following statement is correct? [GATE 2010]

(a) z = 0 is a branch point (b) z = 0 is a pole of order one

(c) z = 0 is a removable singularity (d) z = 0 is an essential singularity

Common data for Q. 35 and Q. 36

Consider a function $f(z)=\frac{z\sin z}{(z-\pi)^2}$ of a complex variable z. [GATE 2010]

35. Which of the following statements is TRUE for the function $f(z)$?

(a) $f(z)$ is analytic everywhere. in the complex plane

(b) $f(z)$ has a zero at $z=\pi$

(c) $f(z)$ has a pole oforder 2 at $z=\pi$

(d) $f(z)$ has a simple pole at $z=\pi$

36. Consider a counterclockwise circular contour $|z|=1$ about the origin. The integral $\oint f(z)dz$ over contour is : [GATE 2012]

(a) $-i\pi$ (b) zero (c) $1+2i\pi$ (d) $2i\pi$

37. The value of the integral $\oint_C e^{1/z} dz$ using the contour C of circle with unit radius $|z|=1$, is:

(a) 0 (b) $1=2\pi i$ (c) $1+2\pi i$ (d) $2\pi i$ [GATE 12]

38. For the function $f(z)=\frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $|z|=1$.....

(Your answer should be an integer) [GATE 2013]

39. The value of the integral $\oint_C \frac{z^2}{e^z+1} dz$, where C is the circle $|z|=4$ is : [GATE 2014]

QUANTA

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

40. Consider $w = f(z) = u(x,y) + iv(x,y)$ to be an analytic function in a domain D. Which one of the following options is NOT correct? [GATE 2015]

(a) $u(x,y)$ satisfies Laplace equation in D

(b) $v(x,y)$ satisfies Laplace equation in D

(c) $\int_{z_1}^{z_2} f(z)dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) $f(z)$ can be Taylor expanded in D

41. Consider a coinptex function $f(z) = \dfrac{1}{z\left(z+\frac{1}{2}\right)\cos(z\pi)}$. Which nne of the following statements is correct? (GATE 2015]

(a) $f(z)$ has simple poles at z = 0 and $z = -\dfrac{1}{2}$

(b) $f(z)$ has a second order pole- at $z = -\dfrac{1}{2}$

(c) $f(z)$ has infinite number of second order poles

(d) $f(z)$ has all simple poles

42. Which of the following is an analytic function of z everywhere in the complex plane? [GATE 2016]

(a) z^2 (b) $\left(z^*\right)^2$ (c) $|z|^2$ (d) $\sqrt{z}$

43. If $z = x + iy$ then the function $f(x,y) = (1+x+y)(1+x-y) + a\left(x^2 - y^2\right) - 1 + 2iy(1 - x - ax)$ where a is real parameter, is analytic in the complex z plane if a is equal to :

(a) –1 (b) +1 (c) 0 (d) i [GATE 2013]

44. The integral $\int_0^{\infty} \dfrac{dx}{4+x^4}$ evalautes to :

(a) π (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{8}$

45. The integral $\int_0^{2\pi} \dfrac{d\theta}{1 - 2a\cos\theta + a^2}$ where $0 < a < 1$, evaluates to : [TIFR 2015]

QUANTA

(a) $\frac{2\pi}{1-a^2}$ (b) $\frac{2\pi}{1+a^2}$ (c) 2π (d) $\frac{4\pi}{1-a^2}$

46. The value of the integral $\oint_C \frac{\sin z}{z^6} dz$, where C is the circle with centre z = 0 and radius 1 unit : [TIFR 2017]

(a) $i\pi$ (b) $\frac{i\pi}{120}$ (c) $\frac{i\pi}{60}$ (d) $-\frac{i\pi}{6}$

47. The value of the integral $\int_0^\infty \frac{dx}{x^4+4}$, is : [TIFR 2017]

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

48. The value of integral $\int_0^\infty \frac{\ln x}{\left(x^2+1\right)^2} dx$ is : [JEST 2012]

(a) 0 (b) $-\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

49. Compute $\lim_{z\to\infty} \frac{\mathrm{Re}\left(z^2\right)+lm\left(z^2\right)}{z^2}$ [JEST 2013]

(a) the limit does not exist (b) 1

(c) –i (d) –1

50. The value of integral

$$I = \oint_c \frac{\sin z}{2z-\pi} dz$$

with c a is circle $|z| = 2$, is [JEST 2014]

(a) 0 (2) $2\pi i$ (c) πi (d) $-\pi i$

51. The value of limit $\lim_{z\to i} \frac{z^{10}+1}{z^6+1}$ is equal to : [JEST 2014]

(a) 1 (b) 0 (c) –10/3 (d) 5/3

52. Given an analytic function $f(x,y) = \phi(x,y) + i\psi(x,y)$ where $\phi(x,y) = x^2 + 4x - y^2 + 2y$. If C is a constant, then which of the following relations is true? [JEST 2015]

(a) $\psi(x,y) = x^2 y \,|\, 4y + C$ (b) $\psi(x,y) = 2xy - 2x + C$

(c) $\psi(x,y) = 2xy + 4y - 2x + C$ (d) $\psi(x,y) = x^2 y - 2x + C$

QUANTA

53. The value of the integral $\int_0^\infty \frac{\ln x}{\left(x^2+1\right)} dx$ is :

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) π^2 (d) 0

54. Given $i=\sqrt{-1}$, then is :

(a) Purely real (b) Purely imaginary

(c) Of the form $x+iy$ with $x\neq 0, y\neq 0$ (d)Not defined

55. Let z_1 and z_2 be two non-zero complex numbers. If

$$|z_1+z_2|=|z_1|+|z_2|$$

Which of the following is true?

(a) $\mathrm{Re}\left(z_1\bar{z}_2\right)<0$, and $\mathrm{Im}\left(z_1\bar{z}_2\right)=0$ (b) $\mathrm{Re}\left(z_1\bar{z}_2\right)>0$ and $\mathrm{Im}\left(z_1\bar{z}_2\right)=0$

(c) $\mathrm{Re}\left(z_1\bar{z}_2\right)>0$ and $\mathrm{Im}\left(z_1\bar{z}_2\right)>0$ (d) $\mathrm{Re}\left(z_1\bar{z}_2\right)<0$ and $\mathrm{Im}\left(z_1\bar{z}_2\right)<0$

56. If 'z' is a complex number, them we can express $\sin^{-1} z$ as :

(a) $-i\ell n\left(iz\pm\sqrt{1-z^2}\right)$ (b) $-i\ell n\left(z\pm\sqrt{z^2-1}\right)$

(c) $\ell n\left(z+\sqrt{z^2+1}\right)$ (d) $\ell n\left(z+\sqrt{z^2-1}\right)$

57. The value of $\oint_C \frac{dz}{(z-3)}$ if C is the circle $|z-2|=5$ is :

(a) πi (b) $2\pi i$ (c) 0 (d) 2π

58. The complex function $f(z)=e^{\frac{1}{(z-1)^2}}$ has singular point at $z=I$ which is :

(a) a pole of order 1 (b) a pole o forder 2

(c) an isolated essential singularity (d) a non-isolated essential singularity

59. The function $\tan(1/z)$ does not have a pole at z =

(a) $\pi/2$ (b) $2/\pi$ (c) $2/(3\pi)$ (d) $-2/(3\pi)$

60. The real part ofa complex function $f(z)$, analytic in a region is given by $u(x,y)=x^2-y^2$. If the function vainshes at z = 0,the imaginary part of the function is :

(a) $2xy$ (b) $-2xy$ (c) x^2+y^2 (d) y^2-x^2

QUANTA

61. The residue of the function, $f(z)=\dfrac{z}{(2z+1)(5-z)}$ at $z=-1/2$ is :

(a) 1/11 (b) –1/11 (c) 1/22 (d) –1/22

62. Two complex numbers Z_1 and Z_2 are given to satisfy $|Z_1|=|Z_2|$ necessarily follows that:

(a) $Z_1=Z_2$ (b) $Z_1=e^{i\phi}Z_2$ (c) $Z_1=Z_2$ (d) $Z_1=Z_2^{-1}$

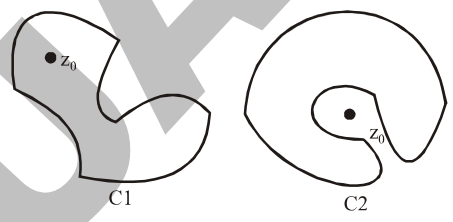
63. Two functions of a complex vatiable $z=x+iy$ are given as (i) $|z|$ and (ii) z^2, which of the following statements is correct?

(a) Both (i) and (ii) are analytic
(b) (i) is analytic and (ii) is nonanalytic
(c) Both are nonanalytic
(d) (i) is nonanalytic and (ii) is analytic

64. A function $f(Z)$ of a complex vartiable $f(Z)$ is given as $f(Z)=Z\exp\left(\dfrac{1}{Z}\right)$. At $Z=0$

(a) It has no singularity
(b) It has a simple pole and residue is 1
(c) It has an essential sigulatity and residue is 1/2
(d) It has a pole of order 2 and residue 1/6

65. A function $f(Z)$ of a complex variable Z is analytic everywhere in the complex plane except at $Z=Z_0$. Two contours C1 and C2 around Z_0 are shown in the figure.



The values of the integrals $I_1=\oint_{C1}\dfrac{f(Z)dZ}{Z-Z_0}, I_2=\oint_{C2}\dfrac{f(Z)dZ}{Z-Z_0}$ are :

(a) $I_1=0, I_2=2\pi i f(Z_0)$ (b) $I_1=2\pi i f(Z_0), I_2=0$

(c) $I_1=\pi i f(Z_0), I_2=\pi i f(Z_0)$ (d) $I_1=2\pi i f(Z_0), I_2=2\pi i f(Z_0)$

66. The value o f the integral $\oint_C \dfrac{z^3}{(z-1)^2}dz$, where C is the circle $|z|=4$ is :

(a) $6\pi i$ (b) $-6\pi i$ (c) $2\pi i$ (d) Zero

67. The leading term in Laurent expansion of $f(z)=\dfrac{1}{z(1-z)^2}$ around z = 0 is :

QUANTA

(a) $\frac{1}{z^3}$ (b) $\frac{1}{z^2}$ (c) $\frac{1}{z}$ (d) 1

68. The value of $\oint_c \frac{e^2(z+3)}{z-2}$ where c is a unit circle with centre at origin is :

(a) $5e^2$ (b) $\frac{5}{2\pi i}e^2$ (c) 0 (d) $10\pi i e^2$

69. If $f(z)=\frac{Z^2+a^2-2aZ}{(Z-a)^3(Z-b)}$ then $Z=a$ is :

(a) Pole of order 1 (b) Pole of order 2

(c) Pole of order 3 (d) An essential singularity

70. Which of the of the following functions is not analytic?

(a) $f(z)=|z|$ (b) $f(z)=z^2$ (c) $f(z)=\cos z$ (d) $f(z)=\sin z$

71. The value of the integral $\oint_c \frac{\cos \pi z}{z(z+1)}$ where C is the circle $|z-1|=3$ is :

(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$

ANSWER-KEY

1.(c)	2.(b)	3.(b)	4.(b)	5.(b)	6.(a)	7.(b)
8.(a)	9.(d)	10.(a)	11.(b)	12.(c)	13.(a)	14.(c)
15.(b)	16.(a)	17.(b)	18.(c)	19.(c)	20.(a)	21.(b)
22.(b)	23.(a)	24.(d)	25.(b)	26.(c)	27.(b)	28.(a)
29.(b)	30.(a)	31.(b)	32.(c)	33.(d)	34.(c)	35.(d)
36.(b)	37.(d)	38.(3)	39.(c)	40.(c)	41.(b)	42.(a)
43.(a)	44.(d)	45.(a)	46.(c)	47.(d)	48.(b)	49.(a)
50.(c)	51.(d)	52.(c)	53.(d)	54.(a)	55.(b)	56.(a)
57.(b)	58.(c)	59.(a)	60.(a)	61.(d)	62.(b)	63.(d)
64.(c)	65.(b)	66.(a)	67.(c)	68.(c)	69.(a)	70.(a)
71.(d)						

QUANTA

SECTION : DEFERENTIAL EQUATION & SPECIAL FUNCTIONS

1. Let $p_n(x)$ (where n = 0,1,2, ...) be ploynomiat of order 'n' with real co-efficients defined in the interval $2 \le x \le 4$. If $\int_2^4 p_m(x)\,p_n(x)\,dx = \delta_{mn}$, then [CSIR June 2011]

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

2. The genarating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x)t^n$ for Legendre polynomials $P_n(x)$ is: $F(x,t) = \left(1-2xt+t^2\right)^{-1/2}$. The value of $P_3(-1)$ is: [CSIR Dec. 2011]

(a) 5/2 (b) 3/2 (c) 1 (d) –1

3. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$. Let, $w(t) = x_1(t)\frac{dx_2(t)}{dt}, x_2(t)\frac{dx_1(t)}{dt}$. If $w(0) = 1$ then $w(0) = 1$ is given by [CSIR Dec. 2011]

(a) 1 (b) e^2 (c) $1/e$ (d) $1/e^2$

4. Let $y(x)$ be a continuous real function in the range of 0 to 2π, satisfying the inhomogeneous differential equation: $\sin x\frac{d^2y}{dx^2} + \cos x\frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of $\frac{dy}{dx}$ at the point $x = \frac{\pi}{2}$. [CSIR June 2012]

(a) is continuous (b) has a discontinuity of 3

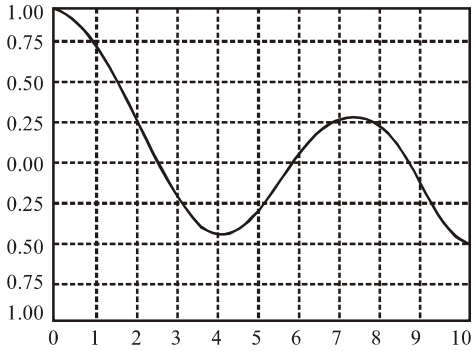
(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

5. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2} - (3-4i)f = 0$ and satisfies the condition $f(0) = 1$ and $f(x) \to 0$ as $x \to \infty$. The value of $f(\pi)$ is : [CSIR June 2012]

(a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi}$

6. The graph of the function $f(x)$ shown below is best described by : [CSIR Dec. 2012]

QUANTA



(a) The bessel function $J_0(x)$ (b) $\cos x$

(c) $e^{-x}\cos x$ (d) $\dfrac{\cos x}{x}$

7. Given that $\sum_{n=0}^{\infty} H_n(x)\dfrac{t^n}{n!} = e^{-t^2+2tx}$, the value of $H_4(0)$ is [CSIR June 2013]

(a) 12 (b) 6 (c) 24 (d) –6

8. The solution of the differential equation $\dfrac{dx}{dt} = x^2$ with the initial condition $x(0)=1$ will blow up as *t* tend to : [CSIR June 2013]

(a) 1 (b) 2 (c) 1/2 (d) ∞

9. Consider the differential equation : [CSIR June 2014]

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

with intial conditions $x(0)=0$ and $x(0)=1$. The solution $x(t)$ attains its maximum value when 't' is :

(a) 1/2 (b) 1 (c) 2 (d) ∞

10. Given $\sum_{n=0}^{\infty} P_n(x)t^n = \left(1-2xt+t^2\right)^{-1/2}$ for $|t|<1$, the value of $P_5(-1)$ is : [CSIR Dec. 2014]

(a) 0.26 (b) 1 (c) 0.5 (d) 1

11. The function $f(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{n!(n+1)!(2)}\left(\dfrac{x}{2}\right)^{2n+1}$ satisfies the differential equation [CSIR Dec. 2014]

(a) $x^2\dfrac{d^2f}{dx^2} + x\dfrac{df}{dx} + \left(x^2+1\right)f = 0$ (b) $x^2\dfrac{d^2f}{dx^2} + 2x\dfrac{df}{dx} + \left(x^2-1\right)f = 0$

QUANTA

(c) $x^2\frac{d^2f}{dx^2}+x\frac{df}{dx}+\left(x^2-1\right)f=0$ (d) $x^2\frac{d^2f}{dx^2}-x\frac{df}{dx}+\left(x^2-1\right)f=0$

12. Consider the differential equation $\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=0$. If $x=0$ at $t=0$ and $x=1$ at $t=1$, the value of x at $t=2$, will be : [CSIR June 2015]

(a) e^2+1 (b) e^2+e (c) $e+2$ (d) $2e$

I3. The solution of the differential equation $\frac{dx}{dt}=2\sqrt{1-x^2}$, with initial condition x = 0 at t = 0 is : [CSIR Dec. 2015]

(a) $x=\begin{cases}\sin 2t, & 0\le t\le\frac{\pi}{4}\\ \sinh 2t, & t\ge\frac{\pi}{4}\end{cases}$ (b) $x=\begin{cases}\sin 2t, & 0\le t<\frac{\pi}{4}\\ 1, & t\ge\frac{\pi}{2}\end{cases}$

(c) $x=\begin{cases}\sin 2t, & 0\le t<\frac{\pi}{4}\\ 1, & t\ge\frac{\pi}{4}\end{cases}$ (d) $x=1-\cos 2t, t\ge 0$

14. The Hermite polynomial $H_n(x)$ satisfies the differential equation :

$$\frac{d^2H_n}{dx^2}-2x\frac{dH_n}{dx}+2nH_n(x)=0$$ [CSIR Dec. 2015]

The corresponding generating function $G(t,x)=\sum_{n=0}^{\infty}\frac{1}{n!}H_n(x)t^n$ satisfies the equation :

(a) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}+2t\frac{\partial G}{\partial t}=0$ (b) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}-2t^2\frac{\partial G}{\partial t}=0$

(c) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}+2\frac{\partial G}{\partial t}=0$ (d) $\frac{\partial^2G}{\partial x^2}-2x\frac{\partial G}{\partial x}+2\frac{\partial^2G}{\partial x\partial t}=0$

15. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the ball experiences a damping force of the form $-\gamma v$, where v is its instantaneous velocity and γ is a constant. Given the values $m=10kg/s$ and $g=10m/s^2$, the distance travelled (in metres) in time t in seconds, is [CSIR Dec. 2016]

(a) $10\left(t+1-e^{-t}\right)$ (b) $10\left(t-1+e^{-t}\right)$ (c) $5t^2-\left(1-e^{t}\right)$ (d) $5t^2$

QUANTA

16. Consider the linear differential equation $\frac{dy}{dx} = xy$. If $y = 2$ at $x = 0$, then the value of y at $x = 2$ is : [GATE 2016]

(a) e^{-2} (b) $2e^{-2}$ (c) e^{2} (d) $2e^{2}$

17. A function $y(z)$ satisfies the ordinary differential equation $y + \frac{1}{2}y - \frac{m^2}{z^2}y = 0$, where m = 0,1,2,3.....

Consider the four statements P, Q, R, S as given below: [GATE 2015

P : z^m and z^{-m} are linearly independent solutions for all values of m.

Q : z^m and z^{-m} are linearly.independent solutions for all values of m > 0.

R : In z and 1 are linearly independent solutions for rn = 0.

S : z^m and ln z are linearly independent solutions for all values of m.

The correct option for the combination of valid statements is

(a) P, R and S only (b) P and R only (c) Q and R only (d) R and S only

18. The solution of the differential equation $\frac{d^2y}{dt^2} - y = 0$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$ is : [GATE 2014]

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

19. The solutions to the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of : [GATE 2011]

(a) Circles with different radii (b) Circles with different centres

(c) Straight tines with different slopes

(d) Straight lines with different intercepts on the y-axis

20. The solution of the differential equation for $y(t) = \frac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions $y(0) = 0$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$ is : [GATE 2010]

(a) $\frac{1}{2}\cosh(t) + t\sinh(t)$ (b) $-\sinh(t) + t\cosh(t)$

(c) $t\cosh(t)$ (d) $t\sinh(t)$

21. Given the recurrence relation for the Legendre polynomials :

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

QUANTA

Which of the following integrals has a non zero value?

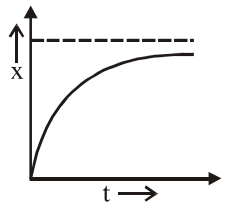
(a) $\int_{-1}^{1} x^2 P_n(x) P_{n+1}(x)\,dx$

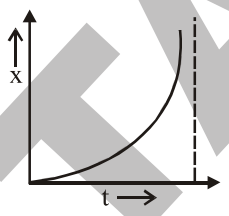
(b) $\int_{-1}^{1} x P_n(x) P_{n+2}(x)\,dx$

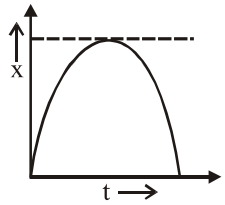
(c) $\int_{-1}^{1} x\left[P_n(x)\right]^2 dx$

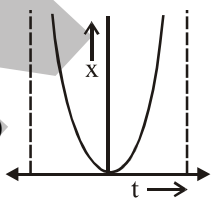
(d) $\int_{-1}^{1} x^2 P_n(x) P_{n+2}(x)\,dx$

22. Which one of the following curves gives the solution of the differential equation $k_1\frac{dx}{dt}+k_2x=k_3$, where k_1, k_2 and k_3 are positive real constants with initial conditions $x=0$ at $t=0$? [GATE 2009]

(a) 

(b) 

(c) 

(d) 

23. Consider the bessel equation:

$$\frac{d^2y}{dz^2}+\frac{1}{z}\frac{dy}{dz}+y=0\,(v=0)$$

Which of the following statements is correct? [GATE 2008]

(a) Equation has regular singular points at $z=0$ and $z=\infty$

(b) Equation has 2 linearly independent solutions that are entire

(c) Equation has an entire solution and a second linearly independent solution is singular at z = 0

(d) Limit $z\to\infty$, taken along X- axis, exists for both the linearly independent solutions

24. The points, where the series solution of the Legendre differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\frac{3}{2}\left(\frac{3}{2}+1\right)y=0$ will diverge, are located at : [GATE 2007]

(a) 0 and 1 (b) 0 and –1 (c) –1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

QUANTA

25. Solution of the differential equation $x\frac{dy}{dx}+y=x^4$, with the boundary condition that $y=1$ at $x=1$, is : [GATE 2005]

(a) $y=5x^4-4$ (b) $y=\frac{x^4}{5}+\frac{4x}{5}$ (c) $y=\frac{4x^4}{5}+\frac{1}{5x}$ (d) $y=\frac{x^4}{5}+\frac{4}{5x}$

Statement for Linked Answer Q. 26 and Q. 27 :

For the differential equaiton $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$ [GATE 2005]

26. One of the solutions is :

(a) e^x (b) $\ln x$ (c) x^{-x^2} (d) x^{x^2}

27. The second linearly independent solution is :

(a) x^{-x} (b) xe^x (c) x^2e^x (d) x^2e^{-x}

28. Consider the differential eqtuation $d^2x/dt^2+2dx/dt+x=0$. At time $t=0$, it is given that $x=1$ and $dx/dt=0$. At $t=1$, the value of x is given by : [GATE 2003]

(a) $1/e$ (b) $2/e$ (c) 1 (d) $3/e$

29. The solution of the differential equation,

$$(1+x)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}-y(x)=0 \text{ is :}$$ [GATE 2003]

(a) Ax^2+B (b) $Ax+Be^{-x}$ (c) $Ax+Be^x$ (d) $Ax+Bx^2$

where A and B are constants.

30. The solution of the system ofdifferential equations,

$$\frac{dy}{dx}=y-z \text{ and } \frac{dz}{dx}=-4y+z$$

is given by (for Aand B are arbitrary constants) [GATE 2001]

(a) $y(x)=Ae^{3x}+Be^{-x};z(x)=-2Ae^{3x}+2Be^{-x}$

(b) $y(x)=Ae^{3x}+Be^{-x};z(x)=2Ae^{3x}+2Be^{-x}$

(c) $y(x)=Ae^{3x}+Be^{-x};z(x)=2Ae^{3x}-2Be^{-x}$

(d) $y(x)=Ae^{3x}+Be^{-x};z(x)=-2Ae^{3x}-2Be^{-x}$

31. The differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$ has the complete solution, in terms of arbitrary constants A and B as : [TIFR 2013]

QUANTA

(a) $A\exp x + Bx\exp x$

(b) $A\exp x + B\exp(-x)$

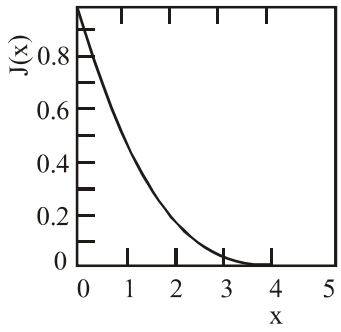
(c) $A\exp x + Bx\exp(-x)$

(d) $x\{A\exp x + B\exp(-x)\}$

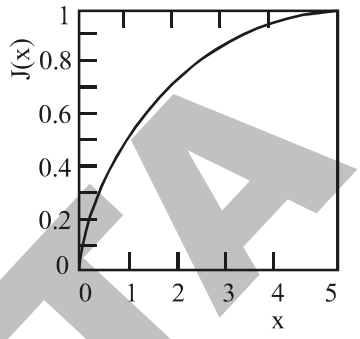
32. The solution of the integral equation $f(x) = x - \int_0^x dt\, f(t)$ has the graphical form :

[TIFR 2014]

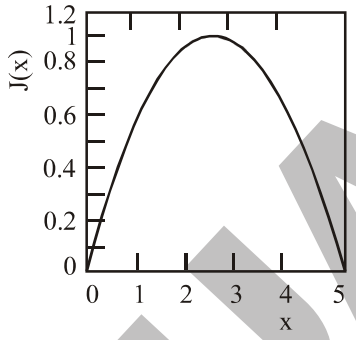
(a)



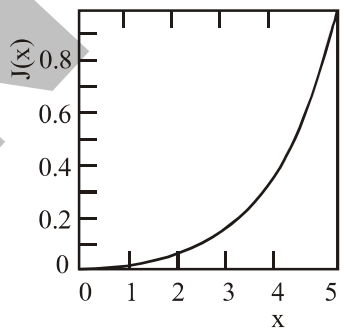
(b)



(c)



(d)



33. Consider the differential equation :

$$\frac{d^2y}{dx^2} = -4\left(y + \frac{dy}{dx}\right)$$

with the boundary conditions that $y(x) = 0$ at $x = \frac{1}{5}$. When plotted as a function of x, for $x \geq 0$, we can say with certainity that the value of y : [TIFR 2015]

(a) first increases, then decreases to zero

(b) first decreases, then increases to zero

(c) has an extremum in the range $0 < x < 1$

(d) oscillates from positive to neagtive with amplitude decreasing to zero

34. The generating function for a set of polynomials in x is given by :

$$f(x,t) = \left(1 - 2xt + t^2\right)^{-1}$$

QUANTA

The third polynomial (order x^2) in this set is [TIFR 2015]

(a) $2x^2+1$ (b) $4x^2+1$ (c) $2x^2-x$ (d) $4x^2-1$

35. The function $y(x)$ satisfies the differential equation $x\frac{dy}{dx}=y(\ln y-\ln x+1)$ with the initial condition $y(1)=3$. What will he the value of $y(3)$? [TIFR 2015]

36. Write down $x(t)$, where $x(t)$ is the solution of the following differential equation :

$$\left(\frac{d}{dt}+2\right)\left(\frac{d}{dt}+1\right)x=1$$

with the boundary conditions $\left.\frac{dx}{dt}\right|_{t=0}=0, x(t)\big|_{t=0}=-\frac{1}{2}$ [TIFR 2017]

37. Consider the differential equation :

$$\frac{dG(x)}{dx}+kG(x)=\delta(x)$$

where k is a constant. Which of the following statement is true? [JEST 2013]

(a) Both G(x) and G'(x) are continuous at x = 0

(b) G(x) is continuous at x = 0 but G'(x) is not

(c) G(x) is discontinuous at x = 0

(d) The continuity properties of G(x) and G'(x) at x = 0 depend on the value of k.

39. What are the solutions to $f''(x)-2f'(x)+f(x)=0$? [JEST 2015]

(a) c_1e^x/x (b) c_1x+c_2/x (c) $c_1xe^x+c_2$ (d) $c_1e^x+c_2xe^x$

39. Given the equations $H'=-aG$ and $G'=bH, G$ can have oscillatory solutions :

(a) $\forall a,b$ (b) for no choice of *a, b*

(c) If $a<0$ and $b>0$ (d) $a<0$ and $b<0$

40. Consider a forced harmonic oscilleator which obeys the differential equation :

$$\frac{d^2y}{dt^2}+y=\sin t\ .$$

Which one of the following is thesolution of the differential equationl with initial condition y (0) = 0?

(a) $y(t)=6\sin t$ (b) $y(t)=16\sin t+\frac{t}{2}\cos t$

(c) $y(t)=12\sin t-\frac{t}{2}\cos t$ (d) $y(t)=6\cos t-\frac{t}{2}\cos t$

QUANTA

41. The solution of the differential equation

$$\frac{dy}{dx}+\frac{2}{3}\frac{y}{x}=0$$

with the condition y = 2 when x = 3 is given by :

(a) $x^2y^3=72$ (b) $x^2y^3=108$ (c) $x^3y^2=108$ (d) $x^3y^2=72$

42. From the solution of the differential equation : $\frac{dy}{dx}=\frac{1}{1+x^2}$

what can you regarding the following series ?

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\ldots$$

(a) The series is divergent (b) The series is absolutely convergent

(c) The series converges to $\frac{\pi}{4}$ (d) The series converges to 0

43. Consider the system of differential equations

$$\frac{dx}{dt}=-y, \qquad \frac{dy}{dt}=x$$

Plotting the various solutions in the x, y-plane one obtains :

(a) Hyperbola (b) Parabola (c) Circles (d) Straight lines

44. If $f(x)=x^4-2=\sum_{n=0}^{\infty}a_nP_n(x)$, then a_3 is :

(a) $\frac{1}{2}$ (b) 1 (c) –1 (d) 0

45. The value of $\frac{J_{1/2}(x)}{J_{-1/2}(x)}$ is :

(a) 0 (b) $\tan x$ (c) $\cot x$ (d) $\tanh x$

46. Legendre's differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

has an ordinary point at x =

(a) –1 (b) 0 (c) 1 (d) ∞

47. For the differential equation: $z^2\frac{d^2y}{dz^2}+z\frac{dy}{dz}+y=0$

QUANTA

(a) $z=\infty$ is an ordinary point

(b) $z=0$ is a regular singular and $z=\infty$ is an irregular singular point

(c) $z=0$ and $z=\infty$ both g-e regular singular points

(d) $z=0$ is an irregular singular point

48. The Legendre polynomial $P_n(x)$, where $-1\le x\le +1$

(a) Is singular at $x=\pm 1$

(b) Satisfies $\int_{-1}^{1} dx P_n^2(x)=1$

(c) Satisfies $\int_{-1}^{1} dx P_n^2(x)=0$ for $n\ge 1$

(d) Is always an even function of x

49. In order to obtain the solution of the initial value problem of the equation ofmotion, $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+16x=0$. How many initial conditions are required?

(a) 0 (b) 1 (c) 2 (d) 3

50. For the Bessel equation, $x^2\frac{d^2x}{dy^2}+x\frac{dy}{dx}+\left(x^2-n^2\right)y=0$, where n is an integer, the maximum number of linearly independent solutions, well-defined at x = 0 is :

(a) zero (b) one (c) two (d) three

51. The general solution of the linear differential equation, $\frac{d^4y}{dx^4}=0$ is, $u(x)$ equal to :

(a) 0

(b) c_3t+c_4

(c) $c_2t^2+c_3t+c_4$

(d) $c_2t^3+c_2t^2+c_3t+c_4$

52. The regular singular points of the associated Legendre differential equationjn the finite domain are

(a) 1, –1 (b) 0 (c) ∞ (d) 2

53. The Wronskian of the linear independent solutions of the differential equation $\frac{d^2y}{dx^2}+4y=0$ is :

(a) Constant (b) 0 (c) ∞ (d) Undetermined

54. Given the Legendre polynormals $P_0(x)=1, P_1(x)=x$ and $P_2(x)\frac{3x^2-1}{2}$ then the polynomial $\left(3x^2+x-1\right)$ can be expressed as :

QUANTA

(a) $P_2(x)-P_1(x)$ (b) $2P_2(x)+P_1(x)$

(c) $P_2(x)+P_1(x)$ (d) $2P_2(x)+P_1(x)+P_0(x)$

55. The solution ofthe differential equation $x\frac{dy}{dx}+y=1$, given the condition y = 0 at x = 1 is:

(a) $y=\left(1-x^2\right)\ell n\,x$ (b) $y=\frac{x^2-1}{x}e^{-x}$ (c) $y=\frac{x-1}{x}$ (d) $y=\sinh(x)$

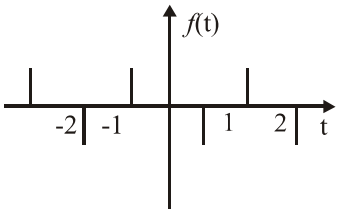
ANSWER-KEY

1.(d)	2.(d)	3.(d)	4.(d)	5.(c)	6.(a)	7.(a)
8.(a)	9.(b)	10.(d)	11.(c)	12.(b)	13.(c)	14.(a)
15.(b)	16.(d)	17.(c)	18.(d)	19.(a)	20.(d)	21.(d)
22.(a)	23.(c,d)	24.(c)	25.(d)	26.(a)	27.(b)	28.(b)
29.(b)	30.(a)	31.(a)	32.(b)	33.(c)	34.(d)	35.(81)
36.()	37.(c)	38.(d)	39.(d)	40.(c)	41.(a)	42.(c)
43.(c)	44.(d)	45.(b)	46.(b)	47.(a)	48.(c)	49.(c)
50.(b)	51.(d)	52.(a)	53.(a)	54.(b)	55.(c)	

QUANTA

SECOND : FOURIER SERIES, FOURIER TRANSFORM, LAPLACE TRANSFORM

1. Consider the periodic function $f(t)$ with time period T as shown hi the fugure below :



[CSIR June 2015]

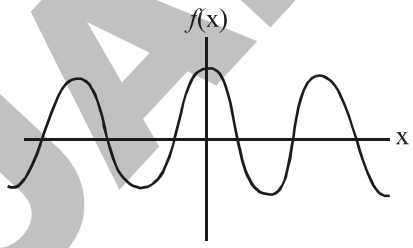
The spikes are located at $t=\frac{1}{2}(2n-1)$, where n = 0, +1, +2.... are Dirac delta functions of strength ± 1. The amplitudes a_n in the fourier expansion $f(t)=\sum_{n=-\infty}^{\infty} a_n e^{i2\pi n/T}$ are given by :

(a) $(-1)^n$ (b) $\frac{1}{n\pi}\sin\frac{n\pi}{2}$ (c) $i\sin\frac{n\pi}{2}$ (d) $n\pi$

2. Fourier Transform of the derivative of the dirac δ function namely $\delta(x)$ is proportional to : [CSIR Dec. 2013]

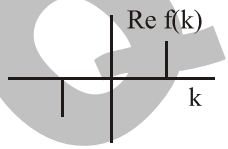
(a) 0 (b) 1 (c) $\sin k$ (d) ik

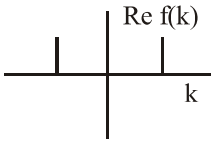
3. The graph of a real periodic function $f(x)$ for range $[-\infty,\infty]$ shown below :

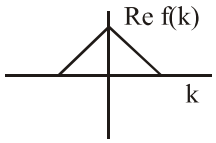


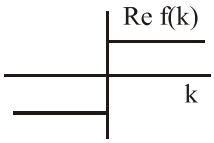
[CSIR June 2014]

Which of the following graphs represents the real part of its Fourier transform?

(a) 

(b) 

(c) 

(d) 

4. The Fourier transform of $f(x)$ is : $f(k)=\int_{-\infty}^{\infty} dx e^{\alpha x} f(x)$. If $f(x)=\alpha\delta(x)+\beta\delta'(x)+\gamma\delta'(x)$ where $\delta(x)$ is the Dirac Delta function (and prime denotes derivative), what is $\tilde{f}(k)$? [CSIR Dec. 2015]

QUANTA

(a) $\alpha + i\beta k + i\gamma k^2$ (b) $\alpha + \beta k - \gamma k^2$ (c) $\alpha - i\beta k - \gamma k^2$ (d) $i\alpha + \beta k - i\gamma k^2$

5. A solution $y(x)$ satisfies the following differential equation :

$$\frac{d^2y}{dx^2} - \omega^2 y = -\delta(x-a)$$

where ω is positive. The Fourier transform $\tilde{f}(k) = \int_{-\infty}^{\infty} dx e^{ikx} f(x)$ of f and the solution of the equation are, respectively [CSIR Dec. 2015]

(a) $\frac{e^{ik\alpha}}{k^2+\omega^2}$ and $\frac{1}{2\omega}\left[e^{-\omega|x-a|} + e^{\omega|x-a|}\right]$ (b) $\frac{e^{ik\alpha}}{k^2+\omega^2}$ and $\frac{1}{2\omega}e^{-\omega|x-a|}$

(c) $\frac{e^{ik\alpha}}{k^2-\omega^2}$ and $\frac{1}{2\omega}\left[e^{-i\omega|x-a|} + e^{i\omega|x-a|}\right]$ (d) $\frac{e^{ik\alpha}}{k^2-\omega^2}$ and $\frac{1}{2\omega}\left[e^{-i\omega|x-a|} - e^{i\omega|x-a|}\right]$

6. What is the Fourier Transform $\int dx e^{ikx} f(x)$ of

$$f(x) = \delta(x) + \sum_{n=1}^{\infty} \frac{d^n}{dx^n}\delta(x)$$

where $\delta(x)$ is the Dirac delta function? [CSIR June 2016]

(a) $\frac{1}{1-ik}$ (b) $\frac{1}{1+ik}$ (c) $\frac{1}{k+i}$ (d) $\frac{1}{k-i}$

7. The graph of the function $f(x) = \begin{cases} 1 & \text{for } 2n < x \le 2n+1 \\ 0 & \text{for } 2n+1 < x \le 2n+2 \end{cases}$ (where n = 0,1,2 .....). Its laplae transform $\tilde{f}(s)$ is : [CSIR Dec. 2011]

(a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s\left(1+e^{-s}\right)}$ (d) $\frac{1}{s\left(1-e^{-s}\right)}$

8. The inverse Laplace tranform of $\frac{1}{s^2(s+1)}$ is : [CSIR June 2013]

(a) $\frac{1}{2}t^2e^{-1}$ (b) $\frac{1}{2}t^2 + 1 - e^{-1}$ (c) $t - 1 + e^{-1}$ (d) $\frac{1}{2}t^2\left(1-e^{-1}\right)$

9. The Laplace transform of $6t^3 + 3\sin 4t$ is : [CSIR June 2015]

(a) $\frac{36}{s^4} + \frac{12}{s^2+16}$ (b) $\frac{36}{s^4} + \frac{12}{s^2-16}$ (c) $\frac{18}{s^4} + \frac{12}{s^2-16}$ (d) $\frac{36}{s^3} + \frac{12}{s^2+16}$

QUANTA

10. The Fourier transform $\int_{-\infty}^{\infty} dx\, f(x) e^{ikx}$ of the function $f(x) = \frac{1}{x^2+2}$ is :

[CSIR Dec. 2016]

(a) $\sqrt{2}\pi e^{-\sqrt{2}|k|}$ (b) $\sqrt{2}\pi e^{-\sqrt{2}k}$ (c) $\frac{\pi}{\sqrt{2}} e^{-\sqrt{2}k}$ (d) $\frac{\pi}{\sqrt{2}} e^{-\sqrt{2}|k|}$

11. The Laplace transform of $f(t) = \begin{cases} \frac{t}{T}, & 0 < t < T \\ 1 & t > T \end{cases}$ is : [CSIR Dec. 2016]

(a) $-\frac{\left(1-e^{-sT}\right)}{s^2 T}$ (b) $\frac{\left(1-e^{-sT}\right)}{s^2 T}$ (c) $\frac{\left(1+e^{-sT}\right)}{s^2 T}$ (d) $\frac{\left(1-e^{sT}\right)}{s^2 T}$

12. A periodic function $f(x) = x$ for $-\pi < x < \pi$ has the Fourier series representation

$$f(x) = \sum_{n=1}^{\infty} \left(-\frac{2}{n}\right)(-1)^n \sin nx$$

Using this, one finds the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$ to be : [GATE 2004]

(a) $2\ln 2$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) $\pi \ln 2$

13. $f(x)$ symmetric periodic function of x i.e. $f(x) = f(-x)$. Then in general, the Fourier series of the function $f(x)$ will be of the form : [GATE 2013]

(a) $f(x) = \sum_{n=1}^{\infty} (a_n \cos nkx + b \sin nkx)$ (b) $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nkx)$

(c) $f(x) = \sum_{n=1}^{\infty} (b_n \sin nkx)$ (d) $f(x) = a_0 + \sum_{n=1}^{\infty} (b_n \sin nkx)$

14. A periodic function $f(x)$ of period 2π is defined in the interval $(-\pi < x < \pi)$ as :

$$f(x) = \begin{cases} -1 & \text{for} -\pi < x < 0 \\ 1 & \text{for} \, 0 < x < \pi \end{cases}$$

[GATE 2016]

The approximate Fourier series expansion for $f(x)$ is :

(a) $f(x) = \frac{4}{\pi}\left[\sin x + \frac{1}{3}\sin 2x + \frac{1}{5}\sin 5x + ...\right]$

QUANTA

(b) $f(x)=\frac{4}{\pi}\left[\sin x-\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x-.....\right]$

(c) $f(x)=\frac{4}{\pi}\left[\cos x+\frac{1}{3}\cos 3x+\frac{1}{5}\cos 5x+.....\right]$

(d) $f(x)=\frac{4}{\pi}\left[\cos x-\frac{1}{3}\cos 3x+\frac{1}{5}\cos 5x-.....\right]$

15. The Heaviside function is defined as :

$$H(t)=\begin{cases}+1 & \text{for } t>0\\ -1 & \text{for } t<0\end{cases}$$

and its fourier transform is given by $-2i/\omega$. The fourier transform of $\frac{1}{2}\left[H\left(t+\frac{1}{2}\right)-H\left(t-\frac{1}{2}\right)\right]$ is : [GATE 2015]

(a) $\frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}$ (b) $\frac{\cos\frac{\omega}{2}}{\frac{\omega}{2}}$ (c) $\sin\frac{\omega}{2}$ (d) 0

16. If the fourier transform $F\left[\delta(x-a)\right]=\exp(-i2\pi va)$, then $F^{-1}(\cos 2\pi va)$ will correspond to : [GATE 2008]

(a) $\delta(x-a)-\delta(x+a)$ (b) a constant

(c) $\frac{1}{2}\left[\delta(x-a)+i\delta(x+a)\right]$ (d) $\frac{1}{2}\left[\delta(x-a)+\delta(x+a)\right]$

17. The ken Fourier component of $f(x)=\delta(x)$ is :

(a) 1 (b) zero (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

18. The Fourier transforin $F(k)$ of a function $f(x)$ is defined as :

$$F(k)=\int_{-\infty}^{\infty}dxf(x)\exp(ikx)$$

then $F(k)$ for $f(x)=\exp\left(-x^2\right)$ is [Given : $\int_{-\infty}^{\infty}\exp\left(-x^2\right)dx=\sqrt{\pi}$] [GATE 2008]

(a) $\pi\exp(-k)$ (b) $\sqrt{\pi}\exp\left(\frac{-k^2}{4}\right)$ (c) $\frac{\sqrt{\pi}}{2}\exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{\pi}\exp\left(-k^2\right)$

19. The Fourier transform of the function $f(x)$ is $F(k)=\int e^{ikx}f(x)dx$. The fourier transform of $df(x)/dx$ is : [GATE 2003]

QUANTA

(a) $dF(k)/dk$ (b) $\int F(k)/dk$ (c) $-ikF(k)$ (d) $ikF(k)$

20. Fourier transform of which of the following functions does not exist? [GATE 2002]

(a) $e^{-|x|}$ (b) xe^{-x^2} (c) e^{x^2} (d) e^{-x^2}

21. Which of the following pairs of thegiven function $F(t)$ and its Laplace transform $f(s)$ is not correct? [GATE 2013]

(a) $F(t)=\delta(t), f(s)=1$ (singularity at 0) (b) $F(t)=1, f(s)=\frac{1}{s}(s>0)$

(c) $F(t)=\sin kt, f(s)=\frac{1}{s^2+k^2}(s>0)$ (d) $F(t)=te^{kt}, f(s)=\frac{1}{(s-k)^2}(s>k, s>0)$

22. If $f(x)=\begin{cases}0 & \text{for } x>3\\ x-3 & \text{for } x\geq 3\end{cases}$ then, the Laplace transform of $f(x)$ is : [GATE 2010]

(a) $s^{-2}e^{3x}$ (b) s^2e^{-3x} (c) s^{-2} (d) $s^{-2}e^{-3x}$

23. If $\tilde{f}(s)$ is the Laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constnat, is [GATE 20051

(a) $\frac{1}{a}\tilde{f}(s)$ (b) $\frac{1}{a}\tilde{f}(s/a)$ (c) $\tilde{f}(s)$ (d) $\tilde{f}(s/a)$

24. The Laplace transform of $f(t)\sin \pi t$ is $F(x)=\frac{\pi}{\left(s^2+\pi^2\right)}, s>0$. Therefore, the Laplace transform of $t\sin \pi t$ is :

(a) $\frac{\pi}{s^2\left(s^2+\pi^2\right)}$ (b) $\frac{2\pi}{s^2\left(s^2+\pi^2\right)^2}$ (c) $\frac{2\pi s}{\left(s^2+\pi^2\right)^2}$ (d) $\frac{2\pi}{\left(s^2+\pi^2\right)^2}$

25. A function $f(x)$ is defined in the range $-1\leq x\leq 1$ by

$$f(x)=\begin{cases}1-x & \text{for } x\geq 0\\ 1+x & \text{for } x<0\end{cases}$$

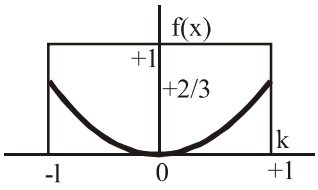
The first few terms in the Fonrier series approximating this function are [TIFR 2010]

(a) $\frac{1}{2}+\frac{4}{\pi^2}\cos \pi x+\frac{4}{9\pi^2}\cos 3\pi x+..$ (b) $\frac{1}{2}+\frac{4}{\pi^2}\sin \pi x+\frac{4}{9\pi^2}\sin 3\pi x+..$

(c) $\frac{4}{\pi^2}\cos \pi x+\frac{4}{9\pi^2}\cos 3\pi x+..$ (d) $\frac{1}{2}-\frac{4}{\pi^2}\cos \pi x+\frac{4}{9\pi^2}\cos 3\pi x+..$

QUANTA

26. The student is asked to find a series approximation for the function $f(x)$ in the domain $-1 \le x \le 1$, as indicated by the thick line in the figure below.



A student represents the function by a sum of three terms

$$f(x) \approx a_0 + a_1 \cos\frac{\pi x}{2} + a_2 \sin\frac{\pi x}{2}$$ [TIFR 2013]

Which of the following would be the best choices for the coefficients a_0, a_1 and a_2 ?

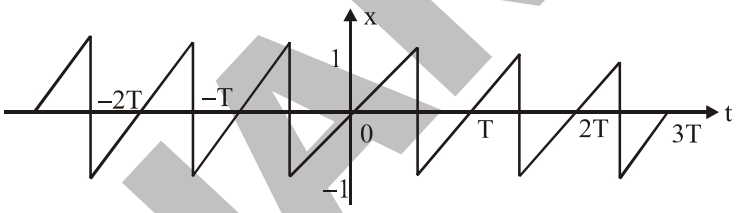
(a) $a_0 = 1, a_1 = -\frac{1}{3}, a_2 = 0$

(b) $a_0 = \frac{2}{3}, a_1 = -\frac{2}{3}, a_2 = 0$

(c) $a_0 = \frac{2}{3}, a_1 = 0, a_2 = -\frac{2}{3}$

(d) $a_0 = -\frac{1}{3}, a_1 = 0, a_2 = -1$

27. Consider the waveform $x(t)$ shown in the diagram below.



The Fourier series for $x(t)$ which gives the closest approximation to this waveform is

(a) $x(t) = \frac{2}{\pi}\left[\cos\frac{\pi t}{T} - \frac{1}{2}\cos\frac{4\pi t}{T} + \frac{1}{3}\cos\frac{3\pi t}{T} + \ldots.\right]$ [TIFR 2017]

(b) $x(t) = \frac{2}{\pi}\left[-\sin\frac{\pi t}{T} + \frac{1}{2}\sin\frac{2\pi t}{T} - \frac{1}{3}\sin\frac{3\pi t}{T} + \ldots.\right]$

(c) $x(t) = \frac{2}{\pi}\left[\sin\frac{\pi t}{T} - \frac{1}{2}\sin\frac{2\pi t}{T} + \frac{1}{3}\sin\frac{3\pi t}{T} + \ldots.\right]$

(d) $x(t) = \frac{2}{\pi}\left[-\cos\frac{\pi t}{T} + \frac{1}{2}\cos\frac{4\pi t}{T} - \frac{1}{3}\cos\frac{6\pi t}{T} + \ldots.\right]$

28. The output intensity *I* of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = -\frac{\omega_0}{Q}I$$

QUANTA

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is : [JEST 2016]

(a) Delta function (b) Gaussian (c) Lorentzian (d) Exponential

ANSWER-KEY

1.(c)	2.(d)	3.(b)	4.(c)	5.(b)	6.(b)	7.(c)
8.(c)	9.(a)	10.(d)	11.(b)	12.(c)	13.(b)	14.(a)
15.(a)	16.(d)	17.(c)	18.(b)	19.(c)	20.(c)	21.(c)
22.(d)	23.(b)	24.(c)	25.(a)	26.(b)	27.(c)	28.(c)

QUANTA

SECTION: PROBABILITY

1. An unbiased dice is thrown three times successively. The probability that the number ofdots on the uppermost surface add up to 16 is [CSIR Dec. 2011]

(a) 1/16 (b) 1/36 (c) 1/108 (d) 1/216

2. A ball is picked at random from one of the two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white? [CSIR

(a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70 June 2012]

3. In a series of five cricket matches, one of the captains calls 'heads' every time when the toss is iken. The probability that he will win 3 times and lose 2 times is. [CSIR Dec 2012]

(a) 1/8 (b) 5/8 (c) 3/16 (d) 5/16

4. Consider three particles A, B and C, each with attribute S that can take two values ±1. Let $S_A = 1, S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to S with probability 1/3. The probability that $S_A + S_B + S_C$ remains unchanged is : [CSIR June 2013]

(a) 2/3 (b) 1/3 (c) 2/9 (d) 4/9

5. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}$ of turning up 1,2,3,4,5 and 6 respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even? [CSIR Dec. 2013]

(a) 144/441 (b) 225/441 (c) 221/441 (d) 220/441

6. A child makes a random walk on a square lattice of lattice constant 'a', taking a step in the north, east, south or west directions with probabilities 0.255, 0.255, 0.245, 0.245 respectively. After a large number of steps N, the expected position of the child w.r.t the strating point is at a distance : [CSIR Dec. 2013]

(a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction

(b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction

(c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction (d) 0

7. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?

[CSIR June 2014]

(a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

8. Let, $y = \frac{1}{2}(x_1 + x_2) - \mu$, where x_1 and x_2 are independent and identically distributed Gaussian random variables of mean μ and standard deviation σ. Then $\frac{\langle y^4 \rangle}{\sigma^4}$ is :

[CSIR June 2014]

QUANTA

(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

9. Two independent random variables m and n, which can take the integer values 0, 1, 2, ... ∞, follow the Poisson distribution, with distinct mean values μ and v respectively. Then [CSIR Dec. 2014]

(a) the probability distribution of the random variable $l = m + n$ is a binomial distribution

(b) the probability distribution of the random variable $r = m - n$ is also a Poisson distribution

(c) the variance of the random variable $l = m + n$ is equal to $\mu + v$

(d) the mean value of the random variable $r = m - n$ is equal to 0

10. A random walker takes a step ofunit length in the positive direction with probabitity 2/3 and a step of unit length in the negative direction with probability 1/3. The mean displacement of the walker after n steps is : [CSIR Dec. 2014]

(a) n/3 (b) n/8 (c) 2n/3 (d) 0

11. Three real variables a, b and c are each randomly chosen from a uniform probability distribution in the interval [0, 1]. The probability that $a + b > 2c$ is [CSIR June 2015]

(a) 3/4 (b) 2/3 (c) 1/2 (d) 1/4

12. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites.The walker starts at the origin and 3 steps. The probability that during this walk no site is visited more than once is : [CSIR Dec. 2015]

(a) 12/27 (b) 27/64 (c) 3/8 (d) 9/16

13. Let X and Y be two independent random variables, each of which follow a normal distribution with same standard deviation σ, but with means μ and $-\mu$ reslaectively. Then the sum $X + Y$ follows a : [CSIR June 2016]

(a) distribution with two peaks at $+\mu$ and mean 0 and standard deviations $\sigma\sqrt{2}$

(b) normal distribution with mean 0 and standard deviations 2σ

(c) distribution with two peaks at $+\mu$ and mean 0 and standard deviation $\sigma\sqrt{2}$

(d) normal distribution with mean 0 and standard deviation $\sigma\sqrt{2}$

14. A box of volume V containing N moleculesofan ideal gas is divided by wall with a hole into two conpartrnents, the volume of the smaller compartment V/3, the variance of the number of particles in it, is [CSIR June 2016]

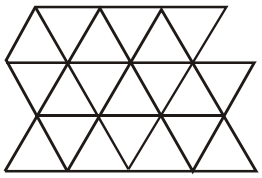
(a) $\frac{N}{3}$ (b) $\frac{2N}{9}$ (c) $\sqrt{N}$ (d) $\frac{\sqrt{N}}{3}$

15. Consider two radioactive atoms, each of which has a decay rate of l per year. The probability that at least one of them decays in the first two years is : [CSIR Dec. 2016]

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $1 - e^{-4}$ (d) $\left(1 - e^{-2}\right)^2$

QUANTA

16. Consider a random walk on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below.



If the probabilities ofmoving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end ofexactly three steps?

[CSIR Dee. 2016]

(a) $\frac{1}{36}$ (b) $\frac{1}{216}$ (c) $\frac{1}{18}$ (d) $\frac{1}{12}$

17. A 100 page book is known to have 200 printing errors distributed randomly through the pages. The probability that one of the pages will be found to be completely free oferrors is closest to : [TIFR 2011]

(a) 67% (b) 50% (c) 25% (d) 13%

18. The probability function for a variable x which assumes only positive values is

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$ [TIFR 2014]

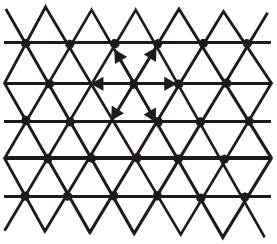
where $\lambda > 0$. The ratio $\langle x \rangle / x$ where $\hat{x}$ is the most probable value and $\langle x \rangle$ is the mean value of the variable x, is

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

19. A random number generator outputs +1 or –1 with equal probability every time it is run. After it is run 6 times, what is the probability that sum of the answers generated is zero? Assume that the individual runs are independent of each other. [TIFR 2015]

(a) 1/2 (b) 5/6 (c) 5/16 (d) 15/32

20. In a triangular lattice a particle moves from a lattice point to any of its 6 neighbouring points with equal probability, as shown in the figure on the right. [TIFR 2016]



The probability that the particle is back at its starting point after 3 moves is :

(a) 5/18 (b) 1/6 (c) 1/18 (d) 1/36

QUANTA

21. An unbiased die is cast twice. The probability that the positive difference (bigger-smaller) between the two numbers is 2 is : [JEST 2012]

(a) 1/9 (b) 2/9 (c) 1/6 (d) 1/3

22. A box contains 100 coins out of which 99 fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one? [JEST 2013]

(a) 0.99 (b) 0.925 (c) 0.075 (d) 0.01

23. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to. [JEST 2013]

(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

24. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x-axis. The probability that they meet after n steps is [JEST 2013]

(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

25. If the distribution fitriction of x is $f(x)=xe^{-x/\lambda}$ over the interval $0<x<\infty$, then mean value of x is : [JEST 2013]

(a) λ (b) 2λ (c) $\lambda/2$ (d) 0

26. If two ideal dice are rolled once, what is the probability of getting atleast one 6? [JEST 2015]

(a) 11/36 (b) 1/36 (c) 10/36 (d) 5/36

27. You receive on average emairs per day during a 365 days. The number of days on average on which you do not receive any emails in that year are : [JEST 2016]

(a) More than 5 (b) More than 2 (c) 1 (d) None of the above

28. The mean value of random variable x with probability density $p(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\left(x^2+\mu x\right)/2\sigma^2\right]$ is : [JEST 2016]

(a) 0 (b) $\mu/2$ (c) $-\mu/2$ (d) σ

ANSWER-KEY

1.(b)	2.(b)	3.(d)	4.(d)	5.(b)	6.(a)	7.(a)
8.(d)	9.(c)	10.(a)	11.(c)	12.(d)	13.(d)	14.(b)
15.(c)	16.(c)	17.(d)	18.(a)	19.(c)	20.(c)	21.(b)
22.(c)	23.(d)	24.(a)	25.(b)	26.(a)	27.(b)	28.(c)

QUANTA

SECTION : MISCELLANEOUS TOPICS

1. Which of the following matrices is an element of the group $SU(2)$? [CS1R June 2011]

(a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$ (c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

2. The character table of C_3, the group of symmetries of an equilateral triangle is given below :

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1f_1$	1	1	b
$2f_2$	1	a	c
$3f_3$	1	1	d

In the above C_1, C_2, C_3 denotes the three classes of C_{3v} containing 1, 3 and 2 elements respectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the free representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v},

(A) The entries a, b, c and d in this table are, respectively.

(a) 2, 1, –1, 0 (b) –1, 2, 0, –1 (c) –1, 1, 0, –1 (d) –1, 1, 1, –1

(B) The reducible representation Γ of C_{3v} with character $\chi = (4,0,1)$ decomposses into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as : [CSIR June 2011]

(a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(1)}$

3. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ? [CSIR June 2912]

(a) $100/\lambda$ (b) $10^4/\lambda^2$ (c) $10^4/\lambda$ (d) $1/\lambda$

4. A bag contains many balls, each with a number painted on it. There are exactly n balls which have tie number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to : [CSIR June 2012]

(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

5. The approximation $\cos\theta = 1$ is valid up to 3 decimal places as long as $|\theta|$ is less than: (take $180°/\pi = 57.29°$) [CSIR June 2013]

QUANTA

(a) 1.28° (a) 1.81° (b) 3.28° (d) 4.01°

6. The solution of the partial differential equation

$$\frac{\partial^2}{\partial t^2}u(x,t)-\frac{\partial^2}{\partial t^2}u(x,t)=0$$

satisfying the boundary conditions $u(0,t)=0=u(L,t)$ and initial conditions $u(x,0)=\sin$ $(\pi x/L)$ and $\frac{\partial}{\partial x}u(x,t)\big|_{t=0}=\sin(2\pi x/L)$ is [CSIR June 2013]

(a) $\sin(\pi x/L)\cos(\pi x/L)+\frac{L}{2\pi}\sin(2\pi x/L)\cos(2\pi x/L)$

(b) $2\sin(\pi x/L)\cos(\pi x/L)-\sin(\pi x/L)\cos(2\pi x/L)$

(c) $\sin(\pi x/L)\cos(2\pi x/L)+\frac{L}{\pi}\sin(2\pi x/L)\sin(\pi x/L)$

(d) $\sin(\pi x/L)\cos(\pi x/L)+\frac{L}{2\pi}\sin(2\pi x/L)\sin(2\pi x/L)$

7. The expression : [CSIR Dec. 2013]

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{\left(x_1^2+x_2^2+x_3^2+x_4^2\right)}$$ is proportional to :

(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $\left(x_1^2+x_2^2+x_3^2+x_4^2\right)^{-3/2}$ (d) $\left(x_1^2+x_2^2+x_3^2+x_4^2\right)^{-2}$

8. Let A. and B be two vectors in three dimensional Euclidean space. Under rotation, the tensor product $T_{ij}=A_iB_j$. [CSIR Dec. 2013]

(a) reduces to a direct sum of three 3 dimesional representations

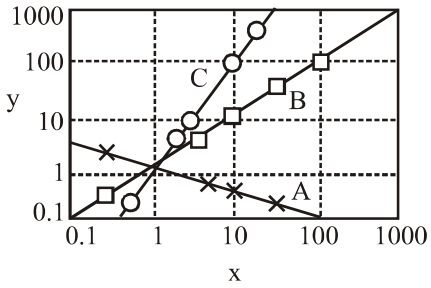
(b) is an irreducible 9 dimensiona representation

(c) reduces to a direct sum of a 1-dimensional a 3-dimensional and a 5-dimensional irreducible representations

(d) reduces to a direct sum of a 1-dimensional and an 5-dimensional irrducible representation

9. 69 three sets of data A, B.and C from an experiment, represented by ×, □, and O, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure. [CS1R Dec. 2013]

QUANTA



The functional dependence $y(x)$ for the sets A, B and C are, respectively.

(a) $\sqrt{x}, x$ and x^2 (b) $-\frac{x}{2}, x$ and $2x$ (c) $\frac{1}{x^2}, x$ and x^2 (d) $\frac{1}{\sqrt{x}}, x$ and x^2

10. The following data is obtained in an expriment that measures the viscosity η as a function ofmolecular weight M for a set of polymers. [CSIR June 2014]

$M(Da)$	$\eta(kPa-s)$
990	0.28 ± 0.03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is :

(a) $\eta - M^{4/9}$ (b) $\eta - M^{3/2}$ (c) $\eta - M^2$ (d) $\eta - M^3$

11. The integral - $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is :

[CSIR June 2014]

(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

12. The function ($\Phi(x,y,z,t) = \cos(z-vt) + \mathrm{Re}(\sin(x+iy))$ satisfies the equation :

[CSIR June 2014]

(a) $\frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2}\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi$ (b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial y^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$ (d) $\left(\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$

13. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix inultiplication? [CSIR Dec. 2014]

(a) $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}$, where $\alpha\beta \neq 1$

QUANTA

(c) $\begin{pmatrix} \alpha & \alpha \\ \beta & \beta \end{pmatrix}$. where $\alpha\beta$ is real

(d) $\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

14. A particle moves in two dimensions on the ellipse $x^2 + 4y^2 = 8$. At a particular instant it is at the point (x, y) = (2, 1) and the x-component of its velocity is 6 (in suitable units). Then the y-component of its velocity is. [CSIR June 2015]

(a) –3 (b) –2 (c) 1 (d) 4

15. The rank-2 tensor x_i, x_j where x_i are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is the elements o f each set transform only into linear combinations of elements in that set) containing : [CSIR June 2015]

(a) 4 and 2 elements (b) 5 and 1 elements

(c) 3, 2 and 1 elements (d) 4, 1 and 1 elements

16. Consider the differential equation $\frac{dy}{dx} = x^2 - y$ with the initial conditiony $y = 2$ at $x = 0$. Let $y_{(1)}$ and $y_{(1/2)}$ be the solutions at $x = 1$ obtained using Euler's forward algorithm with step size 1 and 1/2 respectively. The value of $\left(y_{(1)} - y_{(1/2)}\right) / y_{(1/2)}$ is :

[CSIR June 2015]

(a) $-\frac{1}{2}$ (b) –1 (c) $\frac{1}{2}$ (d) 1

17. Let $f(x,t)$ be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1-dimension. If at $t = 0, f(x,0) = e^{-x^2}$ and $\frac{\partial f}{\partial t}(x,0) = 0$ for all x, then $f(x,t)$ for all future times t > 0 is described by : [CSIR June 2015]

(a) $e^{-\left(x^2 - v^2 t^2\right)}$ (c) $e^{-(x-vt)^2}$

(c) $\frac{1}{4}e^{-(x-vt)^2} + \frac{3}{4}e^{-(x+vt)^2}$ (d) $\frac{1}{2}\left[e^{-(x-vt)^2} + e^{-(x+vt)^2}\right]$

18. In the scattering of some elementary particles, the scattering cross-section a is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of a on these quantities is given by : [CSIR Dec. 2015]

(a) $\sqrt{\frac{hc}{E}}$ (b) $\frac{hc}{E^{3/2}}$ (c) $\left(\frac{hc}{E}\right)^2$ (d) $\frac{hc}{E}$

19. If $y = \frac{1}{\tanh(x)}$, then x is : [CSIR Dec. 2015]

QUANTA

(a) $\ln\left(\frac{y+1}{y-1}\right)$ (b) $\ln\left(\frac{y-1}{y+1}\right)$ (c) $\ln\sqrt{\frac{y-1}{y+1}}$ (d) $\ln\sqrt{\frac{y+1}{y-1}}$

20. The value of the integral $\int_0^8 \frac{1}{x^2+5}dx$, evaluated using Simpson's $\frac{1}{3}$ rule with $h=2$, is [CSIR Dec. 2015]

(a) 0.565 (b) 0.620 (c) 0.698 (d) 0.736

21. The Gauss hypergeometric function $F(a,b,c;z)$, defined by the factor series series expansion around $z=0$ as

$$F(a,b,c;z)=\sum_{h=0}^{\infty}\frac{a(a+1)...(a+n-1)b(b+1)...(b+n-1)}{c(c+1)..(c+n-1)n!}z^n$$

satisfies the equation relation : [CSIR June 2016]

(a) $\frac{d}{dz}F(a,b,c;z)=\frac{c}{ab}F(a-1,b-1,c-1;z)$

(b) $\frac{d}{dz}F(a,b,c;z)=\frac{c}{ab}F(a+1,b+1,c+1;z)$

(c) $\frac{d}{dz}F(a,b,c;z)=\frac{ab}{c}F(a-1,b-1,c-1;z)$

(d) $\frac{d}{dz}F(a,b,c;z)=\frac{ab}{c}F(a+1,b+1,c+1;z)$

22. Using dimensional analysis, Planck defined a characteristic temperature T_P from powers ofthe gravitiorial constant G, Planck's constant h, Bolti.mann constant k_B, and the speed of light c in vacuum. The expression for T_P is proportional to : [CSIR June 2016]

(a) $\sqrt{\frac{hc^5}{k_B^2G}}$ (b) $\sqrt{\frac{hc^3}{k_B^2G}}$ (c) $\sqrt{\frac{G}{hc^4k_B^2}}$ (d) $\sqrt{\frac{hk_B^2}{Gc^3}}$

23. The integral equation

$$\phi(x,t)=\lambda\int dx'dt'$$

is equivalent to the diffe

Q.23 to Q. 42 Page Missing

43. The sum $\sum_{m=1}^{\infty}\frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to : [JEST 2015]

(a) 9 (b) $\sqrt{99}-1$ (c) $\frac{1}{\left(\sqrt{99}-1\right)}$ (d) 11

QUANTA

44. The sun of the infinite series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$ is : [JEST 2016]

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

45. Given the condition $\nabla^2\Phi=0$, the solution of the equation $\nabla^2\Psi=k\vec{\nabla}\Phi.\vec{\nabla}\Phi$ is given by: [JEST 2016]

(a) $\Psi=k\Phi^2/2$ (b) $\Psi=k\Phi^2$ (c) $\Psi=k\Phi\ell n\Phi$ (d) $\Psi=k\Phi\ell n\Phi/2$

ANSWER-KEY

1.(b)	2.(c)	3.(c)	4.(a)	5.(b)	6.(d)	7.(b)
8.(c)	9.(d)	10.(d)	11.(c)	12.(a)	13.(d)	14.(a)
15.(b)	16.(b)	17.(d)	18.(c)	19.(d)	20.(a)	21.(d)
22.(a)	23.(d)	24.(d)	25.(b)	26.(c)	27.(d)	28.(c)
29.(c)	30.(c)	31.(b)	32.(d)	33.(b)	34.(d)	35.(b)
36.(c)	37.(343)	38.(A^n)	39.(a)	40.(c)	41.(d)	42.(d)
43.(a)	44.(d)	45.(a)				

QUANTA